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ECCENTRIC CONNECTIVITY POLYNOMIALS OF AN INFINITE FAMILY OF DENDRIMER

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The eccentricity connectivity polynomial of a molecular graph G is defined as $ECP(G,x)=\sum_{v\in V(G)} deg(v)x^{ecc(v)}$, where $ecc(v)$ is defined as the length of a maximal path connecting v to another vertex of G. In this paper this polynomial is computed for an infinite family of dendrimers.

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1. Introduction

A simple graph $G=(V,E)$ is a finite nonempty set $V(G)$ of objects called vertices together with a (possibly empty) set $E(G)$ of unordered pairs of distinct vertices of G called edges. In chemical graphs, the vertices of the graph correspond to the atoms of the molecule, and the edges represent the chemical bonds.

If $x,y\in V(G)$ then the *distance* $d(x,y)$ between x and y is defined as the length of a minimum path connecting x and y. The *eccentric connectivity index* of the molecular graph G, $\xi^c(G)$, was proposed by Sharma, Goswami and Madan [9]. It is defined as $\xi^c(G)=\sum_{u\in V(G)} deg_G(u)ecc(u)$, where $deg_G(x)$ denotes the degree of the vertex x in G and $ecc(u)=Max\{d(x,u)\mid x\in V(G)\}$, see [5,7] for details. The radius and diameter of G are defined as the minimum and maximum eccentricity among vertices of G, respectively. The eccentric connectivity polynomial of a graph G, $ECP(G,x)=\sum_{v\in V(G)} deg(v)x^{ecc(v)}$, (see [2]). Then the eccentric connectivity index is the first derivative of $ECP(G,x)$ evaluated at $x=1$.

The nanostar dendrimer is a part of a new group of macromolecules that seem photon funnels just like artificial antennas and also is a great resistant of photo bleaching. Recently some people investigated the mathematical properties of this nanostructures in [1,3,4,6,8,10].

We denote the complete graph of order n, the complete bipartite graph with part sizes m,n, the cycle of order n, and the path of order n, by K_n, $K_{m,n}$, C_n, and P_n, respectively.

In Section 2 we compute the eccentricity connectivity polynomial for some specific graphs. In Section 3, we investigate the eccentricity connectivity polynomials for an infinite family of dendrimers.

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2. Eccentricity connectivity polynomial of certain graphs

In this section we consider some specific graphs and compute their eccentricity connectivity polynomials.

Theorem 1 .

1. Let $n \geq 3$. The eccentricity connectivity polynomial of C_n is
$$ECP(C_n, x) = 2nx^{\lfloor \frac{n}{2} \rfloor}.$$
2. Let $n \in \mathbb{N}$. The eccentricity connectivity polynomial of P_{2n} is
$$ECP(P_{2n}, x) = 2x^{2n-1} + 4x^{2n-2} + 4x^{2n-3} + \ldots + 4x^{n+1} + 8x^n.$$
3. Let $n \in \mathbb{N}$. The eccentricity connectivity polynomial of P_{2n+1} is
$$ECP(P_{2n+1}, x) = 2x^{2n} + 4x^{2n-1} + 4x^{2n-2} + \ldots + 4x^n.$$

Proof.

1. It is easy to see that every vertex v of cycle C_n has eccentricity $\lfloor \frac{n}{2} \rfloor$. Also $deg(v) = 2$. So

$$ECP(C_n, x) = \sum_{v \in C_n} 2x^{\lfloor \frac{n}{2} \rfloor} = 2nx^{\lfloor \frac{n}{2} \rfloor}$$

2. It follows from the definition.
3. It follows from the definition.

Theorem 2 .

1. The eccentricity connectivity polynomial of K_n is
$$ECP(K_n, x) = n(n-1)x.$$
2. If $m, n \geq 2$, then the eccentricity connectivity polynomial of $K_{m,n}$ is
$$ECP(K_{m,n}, x) = 2mnx^2.$$

Proof.

1. For every vertex $v \in V(K_n)$, $deg(v) = n-1$ and $e(v) = 1$. Therefore

$$ECP(K_n, x) = \sum_{v \in K_n} (n-1)x = n(n-1)x.$$

2. Suppose that $K_{m,n}$ has partite X and Y, such that $|X| = m$ and $|Y| = n$. Every vertex

$v \in K_{m,n}$, has eccentricity 2 in $K_{m,n}$ if $m,n \geq 2$. So

$$ECP(K_{m,n}, x) = \sum_{v\in X} nx^2 + \sum_{v\in Y} mx^2 = 2mnx^2.$$

3. Eccentric connectivity polynomial of an infinite family of dendrimer

In this section we shall study the eccentric connectivity polynomial of an infinite family of dendrimer.

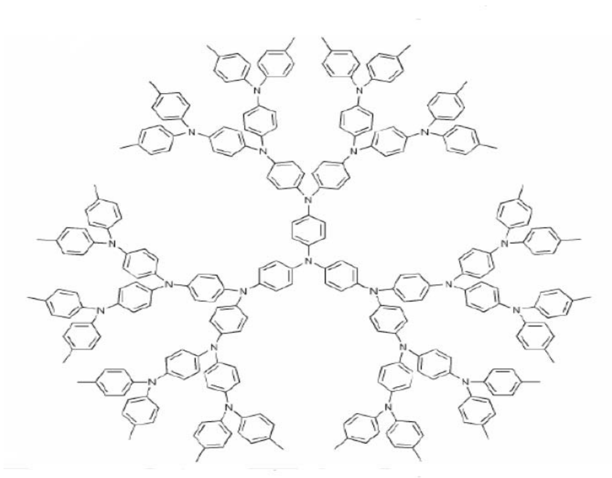

Fig. 1. The first kind of dendrimer of generation 1-3 has grown 3 stages

We compute the chromatic polynomial of the first kind of dendrimer of generation 1-3 has grown n stages. We denote this graph by $D_3[n]$. Figure 1 show the first kind of dendrimer of generation 1-3 has grown 3 stages $(D_3[3])$.

Theorem 3 . *Let $n \in \mathbb{N}$. The eccentric connectivity polynomial of $D_3[n]$ is*

$$ECP(D_3[n], x) = 3\times 2^n x^{10n+10} + 3^2 \times 2^n x^{10n+9} + 12\times 2^n x^{10n+8} + 12\times 2^n x^{10n+7} + 3\times 2^{n-1} x^{10n+6} + 9\times 2^{n-1} x^{10n+5} + 9\times 2^n x^{10n+4} + 6\times 2^{n-1} x^{10n+3} + 6\times 2^{n-1} x^{10n+2} + 9\times 2^{n-1} x^{10n+1} + \ldots + 9\times 2^{1-1} x^{5n+10} + 3^2 x^{5n+9} + 6\times 2x^{5n+8} + 6\times 2x^{5n+7} + 9x^{5n+6} + 3x^{5n+5}.$$

Proof. First we consider the graph $D_3[0]$ as shown in Figure 2.

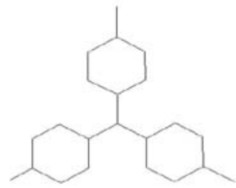

Fig. 2: $D_3[0]$

Suppose that we have the graph $D_3[n-1]$, and we would like to construct $D_3[n]$. In every branch of $D_3[n]$, the graph in Figure 2 added. Since the maximum eccentricity of this added graph is 5, so the eccentricity of previous vertices increase 10. In this step we have $3\times 2^{n-1}$ vertices of kind labeled 1, with eccentricity $5n$. Also there are 3×2^{n} vertices of kind labeled 2 with eccentricity $5n+1$, $3\times 2^{n+1}$ vertices of kind vertex labeled 3 and 4, with eccentricity $5n+2$ and $5n+3$, respectively. 3×2^{n} vertices of kind vertex labeled 5 with eccentricity $5n+4$. Therefore we have the result by definition of eccentric connectivity polynomial. ■

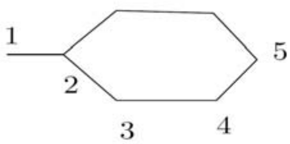

Fig. 3. The added graph in each branch of $D_3[n]$

The following corollary is an immediate consequence of above theorem:

Corollary 1 .

1. The diameter of $D_3[n]$ is $10n+10$.

2. The radius of $D_3[n]$ is $5n+5$.

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