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THE PI AND VERTEX PI POLYNOMIAL OF A T-BENZYL-TERMINATED AMIDE-BASED DENDRIMERS

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Let G = (V, E) be a simple connected graph. Let e = uv ∈ E(G) and $m_u(e)$ be the number of edges closer to u than vand $m_v(e)$ be the number of edges closer to v than u. Then the PI(G, x) of the graph G is defined as $PI(G) = \sum_{e=uv} x^{m_u(e)+m_v(e)}$. We compute the PI vertex PI polynomial of this dendrimer.

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1. Introduction

Let G be a simple connected graph. We denote the vertex set and the edge set of a graph G by V(G) and E(G), respectively. For notation and graph theory terminology not presented here, we follow [1-7].

Let G be a graph and P is a property on G. A counting polynomial for G is a polynomial as $P(G, x) = \sum_k P(G, k)x^k$, where P(G, k) is the frequency of occurrence of the property P of length k and x is simply a parameter to hold k [3].

A topological index of a graph is a number related to a graph which is invariant under graph automorphisms. It is easy to see that every topological index defines a counting polynomial and vice versa. The simplest topological indices are the number of vertices and edges of the graph. The Wiener index (W) is the oldest topological indices [12,13].

Let G be a simple graph. The PI(G) defined as $PI(G) = \sum_{e=uv} m_u + m_v$. Where $m_u(e)$ is the number of edges closer to u than v and $m_v(e)$ is the number of edges closer to v than u[8]. The PI polynomial of G defined as $PI(G, x) = \sum_{e=uv} x^{m_u(e)+m_v(e)}$. It is clear that the derivation of PI(G,x) for x =1 is PI(G).

The $PI_v(G)$ defined as $PI_v(G) = \sum_{e=uv} n_u + n_v$. Where $n_u(e)$ is the number of vertices closer to u than v and $n_v(e)$ is the number of vertices closer to v than u. The PI_v polynomial of G defined as $PI_v(G, x) = \sum_{e=uv} x^{n_u(e)+n_v(e)}$. It is clear that the derivation of $PI_v(G,x)$ for x =1 is $PI_v(G)$[8-13]. Throughout this paper our notation is standard and taken mainly from the standard book of graph theory and [14, 15]. We present some theorems that we need in this paper.

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Theorem A[12]**.** PI(G) = $|E|^2 - \sum_{e=uv} N(e)$ where N(e) = $|\{ xy \mid d(x, e) = d(y, e)\}|$.

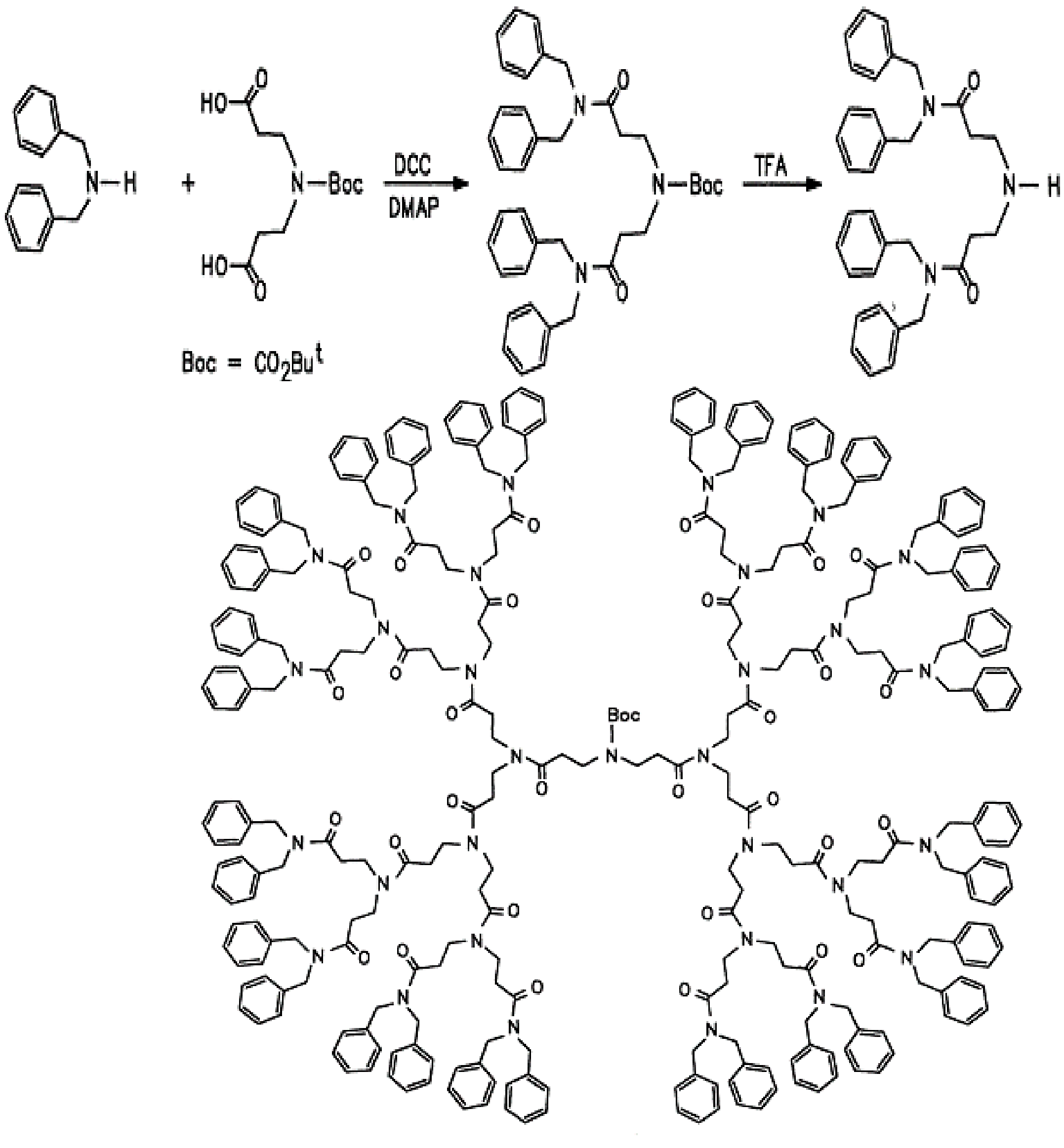
Theorem B. PI(G, x) = $x^{|E|} \sum_{e=uv} x^{-N(e)}$.

Theorem C. $PI_v(G) = |E(G)|.|V(G)| - \sum_{e=uv} N(e)$. Where N(e) is the number of vertices of G with d(x, u) = d(x, v), $x \in V(G)$.

Theorem D. $PI_v(G) \le |E(G)|.|V(G)|$ with equality if and only if G is a bipartite graph.

2. Main results

In this paper, we compute the PI index and PI Polynomial of Dendrimer NS(n) where NS(n) is the following Nano Star.



Construction of *N*-benzyl-terminated amide-based dendrimers.

Lemma. $|V(NS(n)| = 3.2^{n+4} - 8$ and $|E(NS(N)| = 52.2^n – 8$.

Theorem 1. If G is the Nano star NS(n) then PI(NS(n), x) = $6.2^{n+2}x^{52.2^n-10}$ + (28.2^n- 8) $x^{52.2^n-9}$ and $PI(NS(n)) = (52.2^n - 8)(52.2^n - 8 - 1) - 6 \times 2^{n+2}$.

Proof. Let e=uv be an edge on hexagon then $m_u(e) + m_v(e) = m - 2$ = 52.2^n – 8 - 2 = 52.2^n – 10. A simple coputation shows that if e=uv is not an edge on hexagon then we can see that $m_u(e) + m_v(e) = m - 1$= 52.2^n – 9 . Thus

$$PI(NS(n), x) = 6.2^{n+2}x^{52.2^n-10} + (28.2^n- 8)\, x^{52.2^n-9}$$

$$PI(NS(n)) = (52.2^n - 8)(52.2^n - 8 - 1) - 6 \times 2^{n+2}.\square$$

Theorem 2. PI_v(NS(n) ,x) = $x^{3.2^{n+4}-8}$ and PI_v(NS(n)) = (3.2^{n+4} – 8)(52.2^n – 8).

Theorem 3. If G be a connected graph with k disjoint even r-cycle then PI(G) = m^2-m-kr.

Proof. If e be in E(G) then N(e) = 1 otherwise N(e) = 0. By Theorem A PI(G) = m^2-m-kr.

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