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SYNTHESIS OF THE MAXWELL MODEL BASED ON NANOPARTICLES

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The paper studies the viscoelastic fluid described by the synthesized Maxwell model. Assuming the interaction of atoms of the body with embedded nanoparticles leads to a change in model parameters. It is shown that with the addition of nanoparticles with a specific property, it can change the value of the deformation points of a viscoelastic fluid.

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1. Introduction

The establishment of efficient technologies based on nanotechnology in areas such as military equipment, the oil industry, automotive industry, shipbuilding, construction, aviation, etc. is highly appreciated. These technologies are connected with increasing of oil recovery factor of deposits with highly viscous oil. The solution of the problem leads to increasing of volume of hardly extracted oil. There are a lot of researches on this direction.

Among them is the technology based on application of nanoparticles [1- 5]. As it was noted in [2], the efficiency of suggested technology is connected with change of physical mechanical properties of oil containing systems. Taking into account that such systems are described by viscous elastic or viscous plastic models, there is a problem of describing of models with taking into consideration the interactions on atomic or molecular level. For describing of such system without taking into account of nanoparticles was suggested the model of viscous elastic body, that is studied in detail in [7]. For studying of nanoparticles influence it is reasonable to upgrade the existing models of viscous elastic bodies, by bringing in the parameters, characterized the properties of nanosized particles. The building of the models of viscous elastic fluids, containing nanoparticles is the one of urgent problems of nanotechnology.

At the present work is considered the generalized model of viscous elastic fluids [4,7]. It is based on generalization of simple Maxwell model.

2. Setting of the problem

Let us consider the generalized model of viscous elastic fluids that is known as generalized model of Maxwell (pic. 1). It consists of n parallel jointed simple Maxwell models. The simple Maxwell model consists of successively jointed viscous and elastic elements (pic.1).

As it was mentioned above the generalized Maxwell model consists of parallel jointed i-numbered Maxwell models. The simple i- numbered Maxwell model is described by successively

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jointed i - numbered elastic element, for which the following correlation is acceptable $\sigma_{1i} = E_i\,\varepsilon_{1i}$; and i - numbered elastic element for which the following correlation is acceptable $\dot{\varepsilon}_{2i} = \mu_i \sigma_{2i}$.

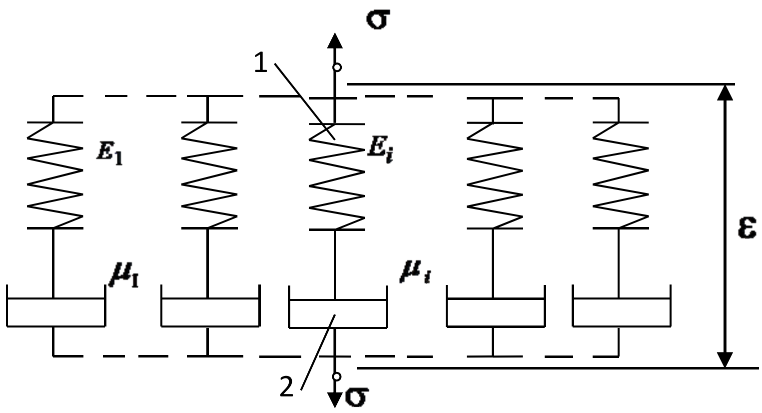

Fig. 1. Generalized Maxwell model Максвелла.

There are following designations: index «1» and «2» correlate to elastic and viscous elements of i - numbered model conformably, σ_{1i}, σ_{2i} and ε_{1i} , ε_{2i} strain and deformation of each above mentioned elements, E_i - elasticity modulus, μ_i - viscosity coefficient of corresponded i - numbered element of simple Maxwell model. The dot demonstrates the derivative on time t .

Let us accept $\sigma_{1i} = \sigma_{2i}$ and designate it σ_i , i.e. $\sigma_{1i} = \sigma_{2i} \equiv \sigma_i$ at consecutive joint in i - numbered simple Maxwell model. Then for generalized model $\sigma = \sum_{i=1}^{n} \sigma_i$, where σ - strain, applied to body, n - number of simple models (pic.1). Then the deformation of i - numbered Maxwell model is defined by correlation

$$\varepsilon_{1i} + \varepsilon_{2i} = \varepsilon_i \qquad (1)$$

Where ε_i - general deformation o fi - numbered Maxwell model (pic.1). Correlation (1) follows from condition of consecutive joint of elastic and viscous elements. It should be noted that parallel joint of elastic and viscous elements the deformation of generalized model is general for all these elements, i.e. $\varepsilon_i = \varepsilon$.

Then accepting $E_i = const$, $\mu_i = const$, ε is:

$$\dot{\varepsilon} = \dot{\varepsilon}_{1i} + \dot{\varepsilon}_{2i} = \frac{1}{E_i}\dot{\sigma}_i + \mu_i \sigma_i \ . \qquad (2)$$

This equation is solved at following initial condition:

$$\sigma_i(t)\big|_{t=0} = E_i \varepsilon(t)\big|_{t=0} , \qquad (3)$$

It means that in start moment the viscous element ‘does not work.

The decision of equation (2) is presented as follow:

$$\sigma_i(t) = \left[E_i \int_0^t \dot{\varepsilon}(\tau) \exp(\mu_i E_i \tau) d\tau + C_{0i} \right] \cdot \exp(-\mu_i E_i t) \qquad (4)$$

Or integrating by parts

$$\sigma_i(t) = E_i \left\{ \varepsilon(t) - \varepsilon(0)\exp(-\mu_i E_i t) - E_i \mu_i \int_0^t \varepsilon(\tau) \exp[-\mu_i E_i (t-\tau)] d\tau \right\} + C_{0i} \exp(-\mu_i E_i t) \qquad (5)$$

Where C_{0i} - the constant of integrating.

Then the decision of equation (2), satisfied to initial condition (3), is as follow:

$$\sigma_i(t) = E_i \left\{ \varepsilon(t) - E_i \mu_i \int_0^t \varepsilon(\tau) \exp[-\mu_i E_i (t-\tau)] d\tau \right\} . \qquad (6)$$

Based on accepted equation and dependence σ from σ_i we get:

$$\sigma(t) = \varepsilon(t) \sum_{i=1}^{n} E_i - \int_0^t \varepsilon(\tau) \sum_{i=1}^{n} E_i^2 \mu_i \exp[-\mu_i E_i (t-\tau)] d\tau \equiv$$

$$\equiv \varepsilon(t) \cdot E_0 - \int_0^t \varepsilon(\tau) \cdot \Gamma(t-\tau) d\tau , \qquad (7)$$

где E_0 - generalized modulus of elasticity, $\Gamma(t-\tau)$ - generalized nuclei of fluidity.

This expression makes able to find dependence of ε and σ. Usually these correlations describe the behavior of viscous elastic “liquid” environments [7]. Some nanotechnological approaches in oil extraction suggest supercharging nanoparticles in oil layer. It leads to change of oil characteristics and consequently to change of parameters describing viscous elastic fluids Assume that structure of initial model does not change by adding nanoparticles i.e. the initial model is generalizing of Maxwell model but with other parameters. Let us define the characteristics of this model. The analysis of the results reveals that adding of nanoparticles leads to change of both elastic and viscous characteristics of the model [4,5]. For its definition let us build the model that makes possible to describe these changes with some approximation.

3. The solving of problem

Assume that elastic element of Maxwell model can be presented as one-dimensional “chain” of atoms, interacting to each other. Such representation is characteristic for polymer molecules.

Let us assume that each embedded nanoparticle takes interstitial place between atoms (though it is possible embedding of more than one nanoparticles). By this the structure of elastic element does not change: it remains rectilinear “chain”. It follows from assumption of preservation of Maxwell model by adding of nanoparticles. Let us assume that particles interact only with neighboring particles. Then without taking into account of nanoparticles it is follow:

$$R_x \sigma_i \approx F_i(a) = F_i'(a) \cdot \Delta a = F_i'(a_0) \cdot a_0 \cdot \varepsilon_i = c_i \cdot a_0 \cdot \varepsilon_i \quad ;$$

where $F_i(a)$ - force of atomic interaction, a - bond length between atoms after deformation, a_0 - before deformation, $c_i = F_i'(a_0)$ - elasticity coefficient, R_x - characteristic size of the square of diametrical section of atom exposed to force, prim – shows the derivative on coordinate, σ_i and ε_i - strain and deformation in each of above mentioned elements.

Then we get the following correlations:

$$\Delta a = a - a_0; \quad \sigma_i = c_i \frac{a_0}{R_x} \varepsilon_i ; \quad \varepsilon_i = \frac{\Delta a}{a_0} .$$

In case with consideration of nanoparticle:

$$R_x \sigma_i = F_{Ni}\left(\frac{a}{2}\right) \approx F'_{Ni}\left(\frac{a_0}{2}\right) \cdot \Delta\left(\frac{a}{2}\right) = F'_{Ni}\left(\frac{a_0}{2}\right) \cdot \frac{a_0}{2} \cdot \varepsilon_i = c_{Ni} \frac{a_0}{2} \cdot \varepsilon_i .$$

From this follows the correlation of elasticity of the “chain” with consideration of nanoparticles.

$$\sigma_i = c_{Ni} \frac{a_0}{2R_x} \varepsilon_i \equiv E_{Ni} \varepsilon_i , \qquad \varepsilon_i = \frac{\sigma_i}{E_{Ni}} , \tag{8}$$

Where «i» and «N» - means that this quantity relates to i - numbered model of Maxwell and nanoparticle. There is assumed that nanoparticle is placed between atoms, in the middle of the “chain”. It should be noted that this location is energetically more profitable.

The viscous element of i - numbered Maxwell model is presented like cylinder, filled by viscous fluid, pressed by piston. Let us assume that nanoparticles are in viscous fluid, it leads to preserving of kind of Maxwell model element. In this case the influence of nanoparticles is analogous to the influence of solid particles in the viscous fluid. The influence of nanoparticles can appear in different ways: increasing of friction force on cylinder walls, change of characteristic properties of fluids, combination of these effects. Besides this it is possible the absence of influence. The choice of effect depends on interrelation of physical chemical properties of fluid and nanoparticles as well as interaction condition. All above mentioned influences of nanoparticles for considered model can be came to change of viscosity coefficient, i.e. it should be introduced the quantity μ_{Ni}. Proceeding from physic effect it follows $\mu_{Ni} \le \mu_i$. Indeed let us consider the first variant. Assume that introduction of nanoparticles in the fluid leads to increasing of friction force of fluid. At tension of the piston with same force the speed of movement decrease due to the friction, i.e.

$$\dot{\varepsilon}_{Ni} = \mu_{Ni} \sigma_i < \dot{\varepsilon}_i = \mu_i \sigma_i ; \quad \mu_{Ni} \le \mu_i .$$

From this follows the reduced inequality.

Then based on correlation (1), definite equation of Maxwell model with consideration of nanoparticles is:

$$\dot{\varepsilon}_i = \frac{\dot{\sigma}_i}{E_{Ni}} + \mu_{Ni} \sigma_i = \frac{\dot{\sigma}_i}{E_i \cdot e_{Ni}} + \mu_i \cdot \upsilon_{Ni} \cdot \sigma_i , \tag{9}$$

There are the next designations:

$$e_{Ni} = \frac{E_{Ni}}{E_i} = c_{Ni} \cdot \frac{1}{2c_i} ; \quad \upsilon_{Ni} = \frac{\mu_{Ni}}{\mu_i} ; \quad (\upsilon_{Ni} \le 1) .$$

The solving of equation (9) by analogues with (4) is:

$$\sigma_i(t) = \left[E_i e_{Ni} \int_0^t \dot{\varepsilon}(\tau) \exp(\mu_i E_i e_{Ni} \upsilon_{Ni} \tau) d\tau + C_{0i} \right] \cdot \exp(-\mu_i E_i e_{Ni} \upsilon_{Ni} t) \tag{10}$$

or

$$\sigma_i(t) = E_i e_{Ni}\left\{\varepsilon(t) - \varepsilon(0)\exp(-\mu_i E_i e_{Ni}\upsilon_{Ni} t) - E_i e_{Ni}\upsilon_{Ni}\mu_i\int_0^t \varepsilon(\tau)\exp\left[-\mu_i E_i e_{Ni}\upsilon_{Ni}(t-\tau)\right]d\tau\right\} +$$
$$+ C_{0i}\exp(-\mu_i E_i e_{Ni}\upsilon_{Ni} t)$$

Because the essence of initial condition does not change, then equation (10), satisfied to initial condition (3), is as follow:

$$\sigma_i(t) = E_i e_{Ni}\left\{\varepsilon(t) - E_i e_{Ni}\upsilon_{Ni}\mu_i\int_0^t \varepsilon(\tau)\exp\left[-\mu_i E_i e_{Ni}\upsilon_{Ni}(t-\tau)\right]d\tau\right\}. \quad (11)$$

Taking into consideration that common strain σ is equal to the sum of strains σ_i, basing on (11), we get:

$$\sigma(t) = \sum_{i=1}^{n}\sigma_i(t) = \varepsilon(t)\sum_{i=1}^{n}E_i e_{Ni} - \int_0^t \varepsilon(\tau)\sum_{i=1}^{n}\left(E_i e_{Ni}\right)^2\upsilon_{Ni}\mu_i\exp\left[-\mu_i E_i e_{Ni}\upsilon_{Ni}(t-\tau)\right]d\tau \equiv$$
$$\equiv \varepsilon(t)E_{0N} - \int_0^t \varepsilon(t)\cdot\Gamma_N(t-\tau)\,d\tau \quad (12)$$

Where E_{0N} - generalized elasticity modulus, $\Gamma_N(t-\tau)$ - generalized nuclei of fluidity with consideration of nanoparticles.

At the constant on time tension load, i.e. in case $\sigma_i(t) = const = \sigma_i(0) > 0$ expression (11) is transformed like follow:

$$\sigma_i(0) = E_i e_{Ni}\left\{\varepsilon(t) - E_i e_{Ni}\upsilon_{Ni}\mu_i\int_0^t \varepsilon(\tau)\exp\left[-\mu_i E_i e_{Ni}\upsilon_{Ni}(t-\tau)\right]d\tau\right\}\times$$
$$\times\left[1+\exp(-\mu_i E_i e_{Ni}\upsilon_{Ni} t)\right]^{-1} \quad (13)$$

Based on obtained equation and dependence of $\sigma(0)$ from $\sigma_i(0)$ we get:

$$\sigma(0) = \varepsilon(t)\sum_{i=1}^{n}E_i e_{Ni}\times\left[1+\exp(-\mu_i E_i e_{Ni}\upsilon_{Ni} t)\right]^{-1} -$$
$$-\int_0^t \varepsilon(\tau)\sum_{i=1}^{n}\left(E_i e_{Ni}\right)^2\upsilon_{Ni}\mu_i\left[1+\exp(-\mu_i E_i e_{Ni}\upsilon_{Ni} t)\right]^{-1}\exp\left[-\mu_i E_i e_{Ni}\upsilon_{Ni}(t-\tau)\right]d\tau$$

So, on the case of generalized Maxwell model has been shown that the presence of nanoparticles in constituent elements leads to change of model parameters. The character of these changes and its values is described in the frame of assuming of nanoparticles interactions with elastic element atoms and viscous fluid.

Let us analyze the influence of nanoparticles in the simple case, when $\mu_{Ni} = \mu_i$ $\left(\upsilon_{Ni} = 1\right)$. Then definition equation is as follow:

$$\varepsilon_{Ni}(t) = \sigma_0\left(\frac{1}{E_i}\frac{1}{e_{Ni}} + \mu_i t\right); \quad \sigma_0 = \sigma(0). \quad (14)$$

Define the relation of deformations, before and after introduction of nanoparticles. It is equal

$$\frac{\varepsilon_{Ni}(t)}{\varepsilon_i(t)} = \left(\frac{1}{E_i}\frac{1}{e_{Ni}} + \mu_i t\right)\cdot\left(\frac{1}{E_i} + \mu_i t\right)^{-1}. \quad (15)$$

The limit values of deformation relations is as follow

$$\frac{\varepsilon_{ni}(t)}{\varepsilon_i(0)} = \frac{1}{e_{Ni}} \; ; \; \frac{\varepsilon_{ni}(t)}{\varepsilon_i(t)} \xrightarrow[t\to\infty]{} 1 .$$

The stationary dot of relation is defined from the soling of following equation:

$$\mu_i\left(\frac{1}{E_i}+\mu_i t\right)^{-1} - \left(\frac{1}{E_i}+\mu_i t\right)^{-2} \mu_i\left(\frac{1}{E_i\, e_{Ni}}+\mu_i t\right) =$$

$$= \mu_i\left(\frac{1}{E_i}+\mu_i t\right)^{-2}\left(\frac{1}{E_i}+\mu_i t - \frac{1}{E_i\, e_{Ni}} - \mu_i t\right) \neq 0$$

It is not difficult to show that the root of this equation does not exist, i.e. the relation of deformations is monotonous function. And at $e_{Ni} > 1$ this function monotonously increases, and at $e_{Ni} < 1$ - monotonously decreases.

So if the elastic bonds of nanoparticles satisfy to following inequality $e_{Ni} < 1$, i.e. $c_{Ni} < 2c_i$, then addition of nanoparticles in viscous elastic fluid leads to increasing of deformation. In controversial case, i.e. if $c_{Ni} > 2c_i$, then addition of nanoparticles leads to decreasing of deformation. So for the model of viscous elastic fluid of Maxwell it is possible to change the value of deformation by addition of definite nanoparticles, i.e. to change the properties of the model.

4. Conclusion

At the work is presented the generalization of the model of theory of viscous elasticity with consideration of nanoparticles. Considering that the view of model is preserved, for generalized Maxwell model is reduced the essential correlations. For this model is shown the influence of nanoparticles on mechanical properties.

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