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OMEGA AND SADHANA POLYNOMIALS OF PERICONDENSED BENZENOID GRAPHS

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The Omega polynomial of a connected graph *G*, denoted by $\Omega(G,x)$, is defined as $\Omega(G,x)=\sum_c m(G,c)x^c$ and the Sadhana polynomial of *G* is defined as $Sd(G,x)=\sum_c m(G,c)x^{|E(G)|-c}$, where *m(G, c)* is the number of strips of length *c* and |*E(G)*| is the number of edges in *G*. In this paper, the Omega and Sadhana polynomials are computed for the pericondensed benzenoid graphs.

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1. Introduction

Let *G* be a graph with vertex set *V(G)* and edge set *E(G)*. For any two vertices *x* and *y* in *V(G)*, the distance between *x* and *y*, denoted by *d(x,y)*, is the length of the shortest path connecting *x* and *y*. Two edges *e=uv* and *f=xy* in *E(G)* are said to be *codistant*, denoted by *e co f*, if *d(x,u)= d(y,v)* and *d(x,v)= d(y,u)= d(x,u)+1= d(y,v)+1*. The relation``*co*''is reflexive, symmetric, but not necessarily transitive. Let $C(e)=\{f\in E(G): f\ co\ e\}$. If the relation ``co'' is transitive on *C(e)*, then *C(e) is* called an *orthogonal cut* ``co'' of the graph *G*.

Let *e=uv and f=xy* be two edges of a graph *G*, which are *opposite* or topological parallel, and this relation is denoted by *e op f*. A set of opposite edges, within the same face or ring, eventually forming a strip of adjacent faces/rings, is called an *opposite* edge *strip ops*, which is a quasi-ortogonal cut *qoc* (*i.e.*, the transitivity relation is not necessarily obeyed). Note that *co* relation is defined in the whole graph while *op* is defined only in a face/ring. We will always assume that the length of *ops* is maximal irrespective of the starting edge. Let *m*(*G*, *c*) be the number of *ops* strips of length *c*.The Omega polynomial of a connected graph *G*, denoted by

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$\Omega(G,x)$, is then defined as [1, 2]: $\Omega(G,x)=\sum_c m(G,c)x^c$ and the Sadhana polynomial of G is defined as [3]: $Sd(G,x)=\sum_c m(G,c)x^{|E(G)|-c}$, where $|E(G)|$ is the number of edges in G. Other recent results concerning the above two computing polynomials can be found in [4-11].

In this paper, the Omega and Sadhana polynomials are computed for a class of pericondensed benzenoid graphs.

2. Main results

In this section, we shall compute the Omega and Sadhana polynomials for the pericondensed benzenoid graphs *B(a, b, c)*, as shown in Fig. 1. Without loss of generality, we may suppose that $a \geq c$ in our following discussions.

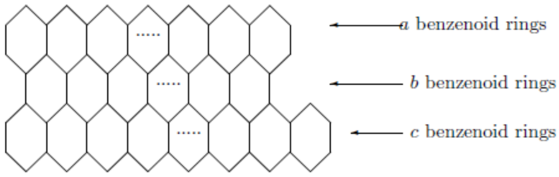

Fig. 1. The benzenoid graph B(a, b, c).

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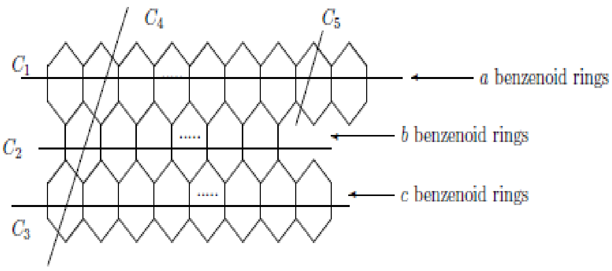

Fig. 2. The benzenoid graph B(a, b, c) with $a \geq c > b$.

Proposition 1. *Let B(a, b, c) be the benzenoid graph as shown in Fig. 2. Then*

$$\Omega(B(a,b,c),x) = x^{a+1}+x^{b+1}+x^{c+1}+2bx^4+(2a+2c-4b)x^2$$

and

$$Sd(B(a,b,c),x) = x^{4a+b+5c+2}+x^{5a+5c+2}+x^{5a+b+4c+2}+2bx^{5a+b+5c-1}+(2a+2c-4b)x^{5a+b+5c+1}.$$

Proof. From Fig. 2 above, one can see that there are exactly five strips in *B(a, b, c)*, namely, C_1,

C_2, C_3, C_4 and C_5. Also, we have $|C_1| = a+1$, $|C_2| = b+1$, $|C_3| = c+1$, $|C_4| = 4$ and $|C_5| = 2$. Consequently, according to the definition of the Omega polynomial, we have

$$\begin{aligned}\Omega(B(a,\, b,\, c), x) &= x^{a+1} + x^{b+1} + x^{c+1} + 2bx^4 + \\ &\quad [2(a-b-1) + 1 + 2(c-b-1) + 1 + 2]x^2 \\ &= x^{a+1} + x^{b+1} + x^{c+1} + 2bx^4 + \\ &\quad (2a + 2c - 4b)x^2\end{aligned}$$

As $|E(G)| = 5a + b + 5c + 3$, we obtain

$$\begin{aligned}Sd(B(a,\, b,\, c), x) &= x^{4a+b+5c+2} + x^{5a+5c+2} + x^{5a+b+4c+2} + \\ &\quad 2bx^{5a+b+5c-1} + (2a + 2c - 4b)x^{5a+b+5c+1}.\end{aligned}$$

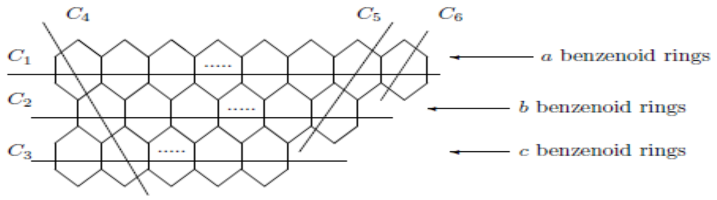

Fig. 3. The benzenoid graph B(a, b, c) with $a > b \geq c$.

Proposition 2. *Let B(a, b, c) be the benzenoid graph as shown in Fig. 3. Then*

$$\begin{aligned}\Omega(B(a,\, b,\, c), x) &= x^{a+1} + x^{b+1} + x^{c+1} + (2c-1)x^4 + \\ &\quad (2b - 2c + 1)x^3 + (2a - 2b + 1)x^2\end{aligned}$$

and

$$\begin{aligned}Sd(B(a,\, b,\, c), x) &= x^{4a+3b+3c+3} + x^{5a+2b+3c+3} + x^{5a+3b+2c+3} + \\ &\quad (2c-1)x^{5a+3b+3c} + (2b - 2c + 1)x^{5a+3b+3c+1} + \\ &\quad (2a - 2b + 1)x^{5a+3b+3c+2}.\end{aligned}$$

Proof. From Fig. 3 above, one can see that there are exactly six strips in *B(a, b, c)*, namely, C_1, C_2, C_3, C_4, C_5 and C_6. Also, we have $|C_1| = a+1$, $|C_2| = b+1$, $|C_3| = c+1$, $|C_4| = 4$, $|C_5| = 3$ and $|C_6| = 2$. So, we have

$$\begin{aligned}\Omega(B(a,\, b,\, c), x) &= x^{a+1} + x^{b+1} + x^{c+1} + [2(c-1) + 1]x^4 + \\ &\quad + [(b - c + 1) + (b - c)]x^3 + [2(a-b-1) + 1 + 2]x^2 \\ &= x^{a+1} + x^{b+1} + x^{c+1} + (2c-1)x^4 + \\ &\quad (2b - 2c + 1)x^3 + (2a - 2b + 1)x^2\end{aligned}$$

As $|E(G)|=5a+3b+3c+4$, we have

$$\begin{aligned} Sd(B(a,\,b,\,c),x) &= x^{4a+3b+3c+3}+x^{5a+2b+3c+3}+x^{5a+3b+2c+3}+ \\ &\quad (2c-1)x^{5a+3b+3c}+(2b-2c+1)x^{5a+3b+3c+1}+ \\ &\quad (2a-2b+1)x^{5a+3b+3c+2}. \end{aligned}$$

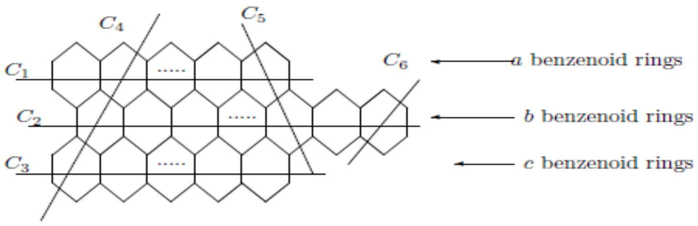

Fig. 4. The benzenoid graph B(a, b, c) with $b \geq a = c$.

Proposition 3. *Let B(a, b, c) be the benzenoid graph as shown in Fig. 4. Then*

$$\begin{aligned} \Omega(B(a,\,b,\,c),x) &= 2x^{a+1}+x^{b+1}+2(a-1)x^4+ \\ &\quad +2x^3+2(b-a+1)x^2 \end{aligned}$$

and

$$\begin{aligned} Sd(B(a,\,b,\,c),x) &= 2x^{5a+5b+4}+x^{6a+4b+4}+2(a-1)x^{6a+5b+1}+ \\ &\quad +2x^{6a+5b+2}+2(b-a+1)x^{6a+5b+3}. \end{aligned}$$

Proof. From Fig. 4 above, one can see that there are exactly six strips in *B(a, b, c)*, namely, C_1, C_2, C_3, C_4, C_5 and C_6. Also, we have $|C_1|=a+1$, $|C_2|=b+1$, $|C_3|=c+1$, $|C_4|=4$, $|C_5|=3$ and $|C_6|=2$. So, we have

$$\begin{aligned} \Omega(B(a,\,b,\,c),x) &= 2x^{a+1}+x^{b+1}+2(a-1)x^4+ \\ &\quad +2x^3+2(b-a+1)x^2 \end{aligned}$$

As $|E(G)|=6a+5b+5$, we have

$$\begin{aligned} Sd(B(a,\,b,\,c),x) &= 2x^{5a+5b+4}+x^{6a+4b+4}+2(a-1)x^{6a+5b+1}+ \\ &\quad +2x^{6a+5b+2}+2(b-a+1)x^{6a+5b+3}. \end{aligned}$$

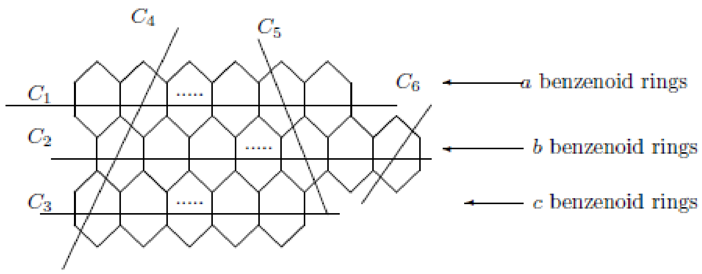

Fig. 5. The benzenoid graph B(a, b, c) with $b \geq a > c$.

Proposition 4. *Let B(a, b, c) be the benzenoid graph as shown in Fig. 5. Then*

$$\begin{aligned}\Omega(B(a,\, b,\, c), x) &= x^{a+1} + x^{b+1} + x^{c+1} + (2c-1)x^4 + \\ &\quad +2(a-c)x^3 + (2b-2a+3)x^2\end{aligned}$$

and

$$\begin{aligned}Sd(B(a,\, b,\, c), x) &= x^{2a+5b+3c+4} + x^{3a+4b+3c+4} + x^{3a+5b+2c+4} + \\ &\quad (2c-1)x^{3a+5b+3c+1} + +2(a-c)x^{3a+5b+3c+2} + \\ &\quad (2b-2a+3)x^{3a+5b+3c+3}.\end{aligned}$$

Proof. From Fig. 5 above, one can see that there are exactly six strips in *B(a, b, c)*, namely, C_1, C_2, C_3, C_4, C_5 and C_6. Also, we have $|C_1| = a+1$, $|C_2| = b+1$, $|C_3| = c+1$, $|C_4| = 4$, $|C_5| = 3$ and $|C_6| = 2$. So, we have

$$\begin{aligned}\Omega(B(a,\, b,\, c), x) &= x^{a+1} + x^{b+1} + x^{c+1} + [2(c-1)+1]x^4 + \\ &\quad +[(a-c+1)+(a-c-1)]x^3 + [2(b-a)+3]x^2 \\ &= x^{a+1} + x^{b+1} + x^{c+1} + (2c-1)x^4 + \\ &\quad +2(a-c)x^3 + (2b-2a+3)x^2\end{aligned}$$

As $|E(G)| = 3a+5b+3c+5$, we have

$$\begin{aligned}Sd(B(a,\, b,\, c), x) &= x^{2a+5b+3c+4} + x^{3a+4b+3c+4} + x^{3a+5b+2c+4} + \\ &\quad (2c-1)x^{3a+5b+3c+1} + +2(a-c)x^{3a+5b+3c+2} + \\ &\quad (2b-2a+3)x^{3a+5b+3c+3}.\end{aligned}$$

By symmetry and Propositions 1--4 above, we obtain our main result of this paper as follows.

Theorem 1. *Let B(a, b, c) be the benzenoid graph as shown in Fig. 1. Then*

$$\Omega(B(a,b,c),x)=\begin{cases} x^{a+1}+x^{b+1}+x^{c+1}+2bx^4+(2a+2c-4b)x^2, & a\geq c>b;\\ x^{a+1}+x^{b+1}+x^{c+1}+2bx^4+(2a+2c-4b)x^2, & c\geq a>b;\\ \quad x^{a+1}+x^{b+1}+x^{c+1}+(2c-1)x^4+(2b-2c+1)x^3 & \\ +(2a-2b+1)x^2, & a>b\geq c;\\ \quad x^{a+1}+x^{b+1}+x^{c+1}+(2a-1)x^4+(2b-2a+1)x^3 & \\ +(2c-2b+1)x^2, & c>b\geq a;\\ \quad x^{a+1}+x^{b+1}+x^{c+1}+(2c-1)x^4+2(a-c)x^3 & \\ +(2b-2a+3)x^2, & b\geq a>c\\ \quad x^{a+1}+x^{b+1}+x^{c+1}+(2a-1)x^4+2(c-a)x^3 & \\ +(2b-2c+3)x^2, & b\geq c>a\\ 2x^{a+1}+x^{b+1}+2(a-1)x^4+2x^3+2(b-a+1)x^2, & b\geq a=c. \end{cases}$$

and

$$Sd(B(a,b,c),x)=\begin{cases} \quad x^{4a+b+5c+2}+x^{5a+5c+2}+x^{5a+b+4c+2}+2bx^{5a+b+5c-1} & \\ +(2a+2c-4b)x^{5a+b+5c+1}, & a\geq c>b;\\ \quad x^{5a+b+4c+2}+x^{5a+5c+2}+x^{4a+b+5c+2}+2bx^{5a+b+5c-1} & \\ +(2a+2c-4b)x^{5a+b+5c+1}, & c\geq a>b;\\ \quad x^{4a+3b+3c+3}+x^{5a+2b+3c+3}+x^{5a+3b+2c+3}+(2c-1)x^{5a+3b+3c} & \\ +(2b-2c+1)x^{5a+3b+3c+1}+(2a-2b+1)x^{5a+3b+3c+2}, & a>b\geq c;\\ \quad x^{3a+3b+4c+3}+x^{3a+2b+5c+3}+x^{2a+3b+4c+3}+(2a-1)x^{3a+3b+5c} & \\ +(2b-2a+1)x^{3a+3b+5c+1}+(2c-2b+1)x^{3a+3b+5c+2}, & c>b\geq a;\\ \quad x^{2a+5b+3c+4}+x^{3a+4b+3c+4}+x^{3a+5b+2c+4}+(2c-1)x^{3a+5b+3c+1} & \\ +2(a-c)x^{3a+5b+3c+2}+(2b-2a+3)x^{3a+5b+3c+3}, & b\geq a>c;\\ \quad x^{3a+5b+2c+4}+x^{3a+4b+3c+4}+x^{2a+5b+3c+4}+(2a-1)x^{3a+5b+3c+1} & \\ +2(c-a)x^{3a+5b+3c+2}+(2b-2c+3)x^{3a+5b+3c+3}, & b\geq c>a;\\ \quad 2x^{5a+5b+4}+x^{6a+4b+4}+2(a-1)x^{6a+5b+1}+2x^{6a+5b+2}+ & \\ 2(b-a+1)x^{6a+5b+3}, & b\geq a=c. \end{cases}$$

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