Analytical study of optical multibistability in an annular cavity laser containing a saturable absorber for inhomogeneous broadening

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In this study, we present a theoretical model that describes accurately the nonlinear phenomenon of optical multistability in an annular cavity laser containing a saturable absorber for inhomogeneous broadening. we develop a simple mathematical model to describe the action of a saturable absorber in a laser cavity (LSA).Using this model, we derive a transcendental equation governing the densities of photons.We theoretically investigate the behaviour of optical bistability (OB) and optical multistability (OM). We determine the densities of photons as function of the pumping of the active medium and analyze the linear stability of the solutions obtained. The approach considered here takes in a phenomenological way into account the essential physical processes which makes it possible to determine the principal parameters in a LSA and their influences on the optical multistability.

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1. Introduction

Optical bistability experienced high interest these last years. Optical bistability belongs to the most important nonlinear optical effects. An essential element of most devices that exhibit optical bistability is an optical resonator, in which a nonlinear material is placed.

The majority of optical flip-flops developed until now utilize two states, which give rise to levels of "high" and "low" energy transmission by the device. In general, optical bistability presenting the phenomenon of hysteresis is obtained as a solution of nonlinear equations.

2. Rate Equations Approach (REA)

From the solution of the Rate Equation Approach (REA) applied to the laser with saturable absorber (LSA) where broadening is inhomogeneous inside a annular vanity in the stationary case, we show the existence of the optical multistability phenomenon while varying the different physical parameters.Then, we complete our study by the analysis of the linear stability of the obtained solutions.

This system consists of the following three equations, one giving the density of photons n_j in the cavity resonator, and two further rate equations determining the variation of the difference of population of the active medium $N_{\mu a}$, and that of the absorbing medium $N_{\mu b}$.

According to the references [1-5], the system of equations is written under the following form:

$$\frac{dn_j}{dt} = -\chi_j n_j + \sum_{\mu} B\, g(\omega_\mu - \omega_j)(n_j + 1)[N_{\mu a} - N_{\mu b}] \quad (1)$$

$$\frac{dN_{\mu a}}{dt} = R_{\mu a} - N_{\mu a}\left[\sum_{j}^{k} B\, g(\omega_\mu - \omega_j) n_j + \gamma_a\right] \quad (2)$$

$$\frac{dN_{\mu b}}{dt} = R_{\mu b} - N_{\mu b}\left[\sum_{j}^{k} B\, g(\omega_\mu - \omega_j) n_j + \gamma_b\right] \quad (3)$$

Where: x_j represent the coefficient of the losses of the resonator for the mode j and B is the coefficient of Einstein; γ_a, γ_b are the damping coefficients of the active medium and absorber, respectively.

We assume $\gamma_b = \xi\gamma_a = \xi\gamma$ with ξ a saturation coefficient: $(0 \le \xi \le 1)$ ω_j *and* ω_μ are respectively the circular frequencies of the mode j and the line of the group of atom μ. $R_{\mu a}$ *and* $R_{\mu b}$ are the pumping rates of the active and the absorbing medium, respectively. Absorption and emission line shapes are supposed top have a Lorentzian shape g with:

$$g(\omega_\mu - \omega_j) = \frac{\Gamma^2}{\Gamma^2 + 4(\omega_\mu - \omega_j)^2}$$

Where Γ is the homogeneous width of line and the function Rμa (ωμ) takes the following form:

$$R_{\mu a} = \frac{R_a \varepsilon^2}{\varepsilon^2 + 4(\omega_j - \omega_0)^2}$$

Where ε is the width of the inhomogeneous line.

▪ Analytical solution of equations

We study here optical bistability and so we are interested only in the case of stationary laser emission. We therefore consider: $\frac{dn_j}{dt} = \frac{dN_{\mu a}}{dt} = \frac{dN_{\mu b}}{dt} = 0$

Starting from equation (2), we obtain:

$$N_{\mu a} = \frac{R_{\mu a}}{\left[\sum_j Bg(\omega_\mu - \omega_j)n_j + \gamma_a\right]} \quad (4)$$

By replacing term $\sum_j^k g(\omega_\mu - \omega_j)$ by the integral $\frac{1}{\pi\Delta\Omega}\int_{-\infty}^{+\infty} \frac{\Gamma^2}{\Gamma^2 + 4(\omega_\mu - \omega_j)} d\omega_j$ in expression (4), we find:

$$N_{\mu a} = \frac{R_{\mu a}}{\gamma\left[Q_J \frac{1}{\pi\Delta\Omega}\int_{-\infty}^{+\infty} \frac{\Gamma^2 d\omega_j}{\Gamma^2 + 4(\omega_\mu - \omega_j)^2} + 1\right]} \quad (5)$$

In order to simplify the calculations, we introduce the following changes of variables:

$Q_j = \frac{B}{\gamma} n_j$, $\quad \frac{B}{\chi_j}\frac{R_a}{\gamma} = \sigma_a$, $\quad \frac{B}{\chi_j}\frac{R_b}{\gamma} = \sigma_b$

with : $j = 0, \pm 1$

By carrying out the integral, equation (5) becomes:

$$N_{\mu a} = \frac{R_{\mu a}}{\gamma\left[\frac{Q_j \Gamma}{2\Delta\Omega} + 1\right]} \quad (6)$$

And

$$N_{\mu b} = \frac{R_{\mu b}}{\gamma\left[\frac{Q_j \Gamma}{2\Delta\Omega} + \xi\right]} \quad (7)$$

By injecting (6) and (7) in equation (1), this one takes the following form:

$$\chi_j n_j = B(n_j + 1)\sum_\mu \frac{\Gamma^2}{\Gamma^2 + 4(\omega_\mu - \omega_j)^2}\left[\frac{R_{\mu a}}{\gamma\left[\frac{Q_j\Gamma}{2\Delta\Omega} + 1\right]} - \frac{R_{\mu b}}{\gamma\left[\frac{Q_j\Gamma}{2\Delta\Omega} + \xi\right]}\right] \quad (8)$$

By replacing the sum ΣRμ (ωμ) by the integral in equation (8), we obtain:

$$\chi_j n_j = \frac{B}{\varepsilon\pi}(n_j + 1)\int_{-\infty}^{+\infty} \frac{\Gamma^2}{\Gamma^2 + 4(\omega_\mu - \omega_j)^2}\left[\frac{R_a \varepsilon^2}{\gamma\left[\varepsilon^2 + 4(\omega_j - \omega_0)^2\right]\left[\frac{Q_j\Gamma}{2\Delta\Omega} + 1\right]} - \frac{R_{\mu b}}{\gamma\left[\frac{Q_j\Gamma}{2\Delta\Omega} + \xi\right]}\right] d\omega_\mu \quad (9)$$

After several mathematical transformations, equation (9) is given by the following formula:

$$\chi_j n_j = \frac{B\Gamma}{2\varepsilon\gamma}(n_j + 1)\left[\frac{R_a \varepsilon^2}{\left[\varepsilon^2 + 4(\omega_j - \omega_0)^2\right]\left[\frac{Q_j\Gamma}{2\Delta\Omega} + 1\right]} - \frac{R_{\mu b}}{\left[\frac{Q_j\Gamma}{2\Delta\Omega} + \xi\right]}\right] \quad (10)$$

We pose: $K = \frac{\varepsilon^2}{\varepsilon^2 + 4k^2\Delta\Omega^2}$ with (ω_j- ω_0) = kΔΩ where (k+1) = nombre of modes considered.

By introducing the various changes into equation (10), we obtain:

$$n_j = \frac{\Gamma}{2\varepsilon}(n_j + 1)\left[\frac{\sigma_a K}{\left[\frac{Q_j\Gamma}{2\Delta\Omega} + 1\right]} - \frac{\sigma_b}{\left[\frac{Q_j\Gamma}{2\Delta\Omega} + \xi\right]}\right] \quad (11)$$

We obtain after simple transformations, a cubic equation in Q_j of the following form:

$$aQ_j^3 + bQ_j^2 + cQ_j + d = 0 \quad (12)$$

Where:

$$a = 1 \;,\quad b = \Delta\Omega\left[\frac{2(1+\xi)}{\Gamma} - \frac{(\sigma_a K - \sigma_b)}{\varepsilon}\right], c = \left[\frac{4\xi\Delta\Omega^2}{\Gamma^2} - \frac{2\Delta\Omega^2}{\varepsilon\Gamma}(\sigma_a K\xi - \sigma_b) - \frac{B\Delta\Omega}{\varepsilon\gamma}(\sigma_a K - \sigma_b)\right]$$

$$and \quad d = -\frac{2B\Delta\Omega^2}{\varepsilon\gamma\Gamma}(\sigma_a K\xi - \sigma_b)$$

The equation (12) is a cubic equation for which the last term tends towards zero ($d << 1$) (the value of B/γ is equal to 10 [-11]). Thus, we can apply a variationnel method. The equation (12) can be put in the following form:

$$\left(Q_j + \frac{d}{c}\right)\left(a\overline{Q}_j^{\,2} + b\overline{Q}_j + c\right) = 0 \qquad (13)$$

The solutions of equation (13) are given by:

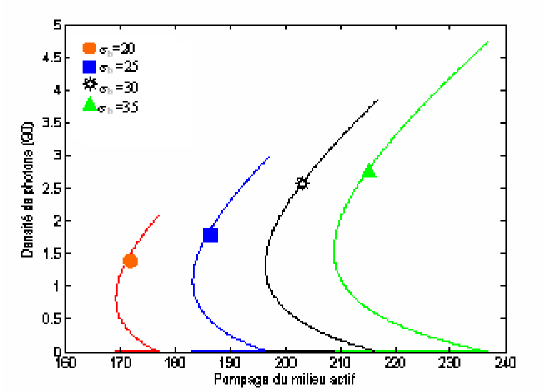
$$Q_{j0} = -\frac{d}{c} \quad \text{and} \quad Q_{j1,2} = \overline{Q}_{j1,2} - \frac{d}{3\overline{Q}_{j1,2}^{\,2} + 2b\overline{Q}_{j1,2} + c} \qquad (14)$$

Where $\overline{Q}_{j1,2} = \dfrac{-b \pm \sqrt{\Delta}}{2}$ with $\Delta = b^2 - 4a\,c$

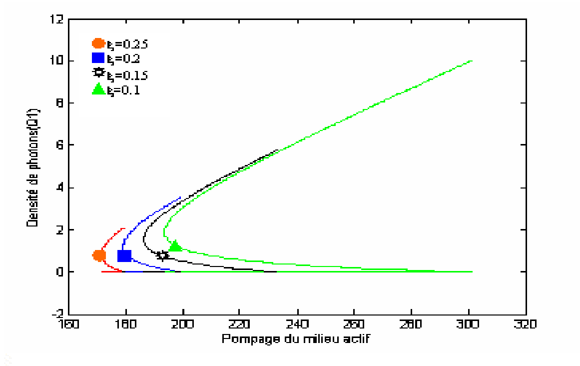
3. Discussion and numerical examples

By laying down certain conditions and solving this equation of the 3 degree, we obtain the results which are represented in the figures below:

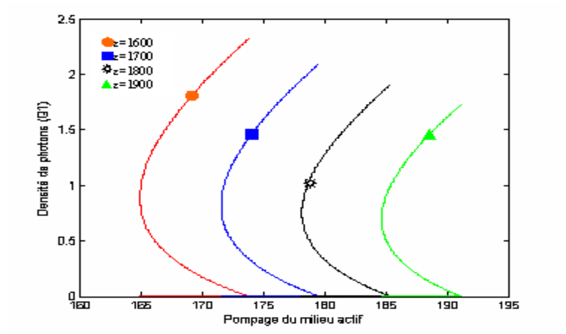
3.1. For the j=0 mode



a) σ_b varies for (ξ=0.25, ε=1700)



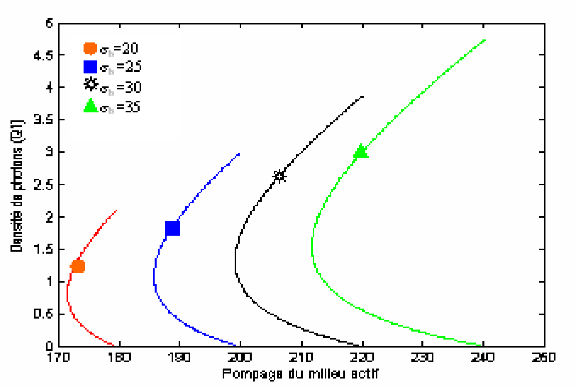
b) ξ varies for (σ_b=20, ε=1700)



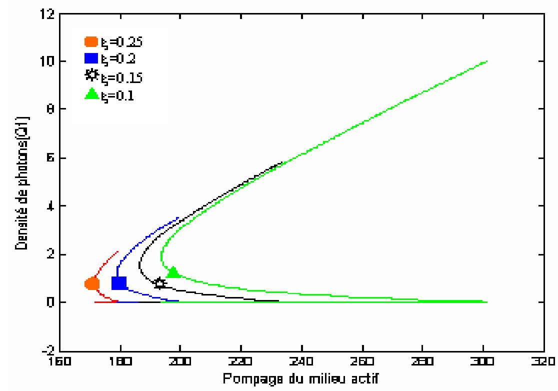
(c) ε varies for (σ_b=20, ξ=0.25).

Fig. 1. Evolution curve of the photons density Q_j as function to the pumping of the active medium (σ_a) whenever the coefficient of the losses is constant for the j=0 mode

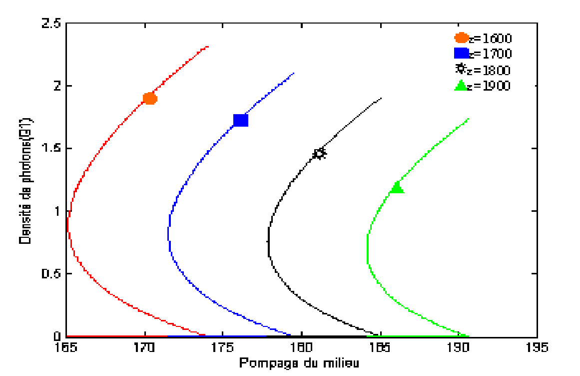
3.2. For the j=1 mode



a) σ_b varies for (ξ=0.25, ε=1700)



b) ξ varies for (σ_b=20, ε=1700).



c) ε varies for (σ_b=20, ξ=0.25)

Fig. 2. Evolution curve of the photons density Q_j as function to the pumping of the active medium (σ_a) whenever the coefficient of the losses is constant for the j=1 mode.

By examining the influence of the parameters of the L.S.A on the appearance of the optical multistability, we notice that a reduction in the coefficient of saturation, leads to an increase in the interval of the density of photons in which the system shows optical multistabitity. This can be explained in the following way: the difference in energy of the absorbing medium must correspond to the frequency of the laser transition in the active medium. In the initial state, the saturable absorber has its maximum of opacity; i.e. the state is not saturated. The luminous irradiation of the saturable absorber at the frequency of oscillation gives rise to a resonant absorption and of spontaneous emission. Then, when the pumping of the absorbing medium σ_b increases, the saturation is increases, too.

After the end of the irradiation, energy relaxation ensures the return to the fundamental state and the absorber becomes again unsaturated. On the other hand, if the pumping of the absorbing medium increases, it causes the saturation of the medium thus and generates an increase in the interval of the effect of the optical bistability and consequently the increase in extended in the density of photons.

Concerning the parameter **ε ,** we notice an increase in the interval of optical bistability as well as of the extent of the density of cavity photons when ε decreases

4. Linearization of the nonlinear system of equations in the vicinity of the stationary solution

The linear analysis of stability consists in determining the evolution of small variations of balances. If the balance is stable, their variations diminish in time. On the other hand, they diverge in the case of an unstable balance. One linearizes the system of differential equations in the vicinity of the stationary solutions and considers for this purpose:

$$\begin{cases} n_j = n_{js} + \Delta n_j(t) \\ N_a = N_{as} + \Delta N_a(t) \\ N_b = N_{bs} + \Delta N_b(t) \end{cases} \quad (15)$$

Where:

n_{js}, N_{as}, N_{bs} are the stationary values

$\Delta n_j, \Delta N_a, \Delta N_b$ are small corresponding variations of quantities with respect to their stationary values.

When replacing n_j, N_a *et* N_b in the equations of the system (1) and neglecting all quantities which are of second order in smallness length of the absorbing medium, one obtains finally a system of linear differential equations for the variations $\Delta n_j(t), \Delta N_a(t)$ *and* $\Delta N_b(t)$ which is given by:

$$\begin{bmatrix} \frac{d\Delta n_j}{dt} \\ \frac{d\Delta N_a}{dt} \\ \frac{d\Delta N_b}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{pmatrix} \Delta n_j \\ \Delta N_a \\ \Delta N_b \end{pmatrix}$$

Where:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (16)$$

Where:

$a_{11} = -\chi_j + \sum_\mu Bg(\omega_\mu - \omega_j)[N_{\mu a} - N_{\mu b}]$, $a_{12} = (n_{js} + 1) \sum_\mu Bg(\omega_\mu - \omega_j)$

$a_{13} = -(n_{js} + 1) \sum_\mu Bg(\omega_\mu - \omega_j)$, $a_{21} = -N_{\mu as} \sum_\mu Bg(\omega_\mu - \omega_j)$

$a_{22} = -\left[\sum_\mu Bg(\omega_\mu - \omega_j) n_{js} + \gamma\right]$, $a_{23} = 0$ *and* $a_{31} = -N_{\mu bs} \sum_\mu Bg(\omega_\mu - \omega_j)$

The calculation of the determinant of $(A + I\lambda)$ gives the characteristic equation of cubic order in λ :

$$\lambda^3 - b_2\lambda + b_1\lambda - b_0 = 0 \quad (17)$$

Where:

$$b_2 = -(a_{11} + a_{22} + a_{33})$$

$$b_1 = a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} - a_{12}a_{21} - a_{13}a_{31}$$

and $b_0 = -(a_{11}a_{22}a_{33} - a_{12}a_{21}a_{33} - a_{13}a_{31}a_{22})$.

λ is the eigenvalue of the matrix and I is the unit matrix of order three.

It is noticed that equation (17) is an algebraic equation of the third degree. To know if the state is stable or unstable, one has to determine the roots of equation (17) .Indeed if the real parts of the four roots of the characteristic equation (17) are positive, the state is stable. If, on the contrary, at least one of these roots has a negative real part, this state will be unstable.

5. Conclusions

In this study, we have developed a theoretical model that describes accurately the nonlinear phenomenon of optical bistability in annular resonator.

It is concluded that for all the stable solutions, therefore we can say that there is the optical multistability under certain conditions. All the branches of hysteresis are stable; this may be due to the inhomogeneous broadening. These results are in good agreement with the tendency of bibliography sources.

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