Effect of harmonic undulator optical klystron scheme on betatron oscillations in free electron laser

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In this paper we discuss the harmonic undulator optical klystron free electron laser to enhance intensity and gain of free electron laser. Inclusion of betatron oscillation of electron while entering the undulator magnets gives extra perturbation which reduces its intensity and gain in free electron laser at several harmonics. In this we have enhanced our intensity and gain with the help of a harmonic undulator optical klystron scheme and compare the results with that of a standard optical klystron free electron laser.

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1. Introduction

The FEL uses a beam of relativistic electrons passing through a periodic, transverse magnetostatic field-the “wiggler” field to amplify the laser light beam propagating along the axis of the electron beam. In the simplest FEL, the undulator field is a planar/helical undulator field. However due to increased need and due to a variety of FEL related applications there had been several novel schemes to design undulator fields. These are variable polarized undulator, bi-harmonic undulator, optical klystron undulator, two frequency undulator, tapered undulator. In addition there had been non-wiggler waveguide FELs such as Smith Purcell FELs and Cerenkov FELs.

An optical klystron consists of two undulators separated either by a drift section or a dispersive magnet[1-8]. The drift section or the dispersive section allows relatively weak optical fields in the first undulator to cause electrons bunch and radiate coherently as they pass through the second undulator. This gives an increase in the free electron laser gain in weak fields for a given interaction length of a conventional free electron laser. The normal OK FEL consists of two sections of undulator separated by a dispersive section. In the dispersive section the electron phase ξ get modulated by $\Delta\xi = \nu D$ where $D = N_d / N_u$ is the dimensionless time of the drift. N_d is the equivalent undulator period in the dispersive section.

Over the years, there have been suggestions on several non-conventional undulator schemes to answer specific problems of free electron laser physics[9-22]. The use of harmonic undulator schemes has been realized as a potential applicant to achieve simultaneous saturated power at the fundamental and at higher harmonics. In this paper we argue the use of harmonic undulators for Klystron free electron laser operation for selective harmonic lasing.

2. Undulator radiation

We assume the electron moves on axis in a two harmonic undulator system whose on-axis field given by,

$$\vec{B} = \left[0, B_0\left\{a_1 \sin(k_u z) + a_2 \sin(k_h z)\right\}\hat{y}, 0\right] \quad (1)$$

where $k_u = 2\pi/\lambda_u$, $k_h = hk_u$, λ_u is the undulator wavelength, B_0 is peak field strength. h is an integer multiple. The trajectory of the electron is determined through the Lorentz equation. This gives

$$x(t) = -\left[\frac{K_1 c}{\gamma\omega_u}\sin(\omega_u t) + \frac{K_h c}{\gamma h\omega_u}\sin(h\omega_u t)\right] \quad (2)$$

where, $K_1 = \dfrac{eB_0 a_1}{m_0 c\omega_u}$, $K_h = K_1\Delta$, $\Delta = (a_2/ha_1)$, $\omega_u = k_u c$, K_1 & K_h define the undulator parameter of the respective fields. The z-motion is,

$$z(t) = \beta^* ct - \frac{cK_1^{\,2}}{8\gamma^2\omega_u}\sin(2\omega_u t) - \frac{cK_h^{\,2}}{8\gamma^2 h\omega_u}\sin(h\omega_u t) - \frac{cK_1K_h}{2\gamma^2\omega_u(1+h)}\sin\omega_u(1+h)t$$
$$-\frac{cK_1K_h}{2\gamma^2\omega_u(1-h)}\sin\omega_u(1-h)t \qquad (3)$$

where,

$$\beta^* = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K_1^{\,2}}{2} + \frac{K_h^{\,2}}{2}\right]$$

The dependence of the field on the transverse coordinates is found from the Maxwell equations. We assume that the field components for Harmonic undulator are given by,

$$B_x(x,y,z) = F_1(x,y)\{a_1\sin(k_u z)\} + F_2(x,y)\{a_2\sin(k_h z)\}$$
$$B_y(x,y,z) = H_1(x,y)\{a_1\sin(k_u z)\} + H_2(x,y)\{a_2\sin(k_h z)\}$$
$$B_z(x,y,z) = G_1(x,y)\{a_1\cos(k_u z)\} + G_2(x,y)\{a_2\cos(k_h z)\} \qquad (4)$$

The quantities $H_{1,2}(x,y), F_{1,2}(x,y), G_{1,2}(x,y)$ have to be evaluated through a set of initial conditions i.e.;

$$H_1(0,0) = B_0\ ,\ F_1(0,0) = 0\ , G_1(0,0) = 0$$
$$H_2(0,0) = -B_0\ ,\ F_2(0,0) = 0\ , G_2(0,0) = 0 \qquad (5)$$

from $\nabla . B = 0$ implies

$$\frac{\partial}{\partial x}F_1(x,y) + \frac{\partial}{\partial y}H_1(x,y) = k_u G_1(x,y)$$
$$\frac{\partial}{\partial x}F_2(x,y) + \frac{\partial}{\partial y}H_2(x,y) = k_h G_2(x,y) \qquad (6)$$

and from $\nabla \times B = 0$ we get,

$$\frac{\partial}{\partial y}G_1(x,y) = k_u H_1(x,y); \frac{\partial}{\partial y}G_2(x,y) = k_h H_2(x,y)$$
$$\frac{\partial}{\partial x}G_1(x,y) = k_u F_1(x,y); \frac{\partial}{\partial x}G_2(x,y) = k_h F_2(x,y)$$
$$\frac{\partial}{\partial x}H_1(x,y) = \frac{\partial}{\partial y}F_1(x,y); \frac{\partial}{\partial x}H_2(x,y) = \frac{\partial}{\partial y}F_2(x,y) \qquad (7)$$

which one combined provide the following eq. For G:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)G_1(x,y) = k_u^{\,2}G_1(x,y)$$
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)H_1(x,y) = k_u^{\,2}H_1(x,y)$$
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)F_1(x,y) = k_u^{\,2}F_1(x,y)$$
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)G_2(x,y) = k_h^{\,2}G_2(x,y)$$
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)H_2(x,y) = k_h^{\,2}H_2(x,y)$$
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)F_2(x,y) = k_h^{\,2}F_2(x,y) \qquad (8)$$

Assuming that

$$G_1(x,y) = G_1(x)G_1(y)$$
$$G_2(x,y) = G_2(x)G_2(y) \qquad (9)$$

We get from eq(8)

$$\frac{d^2}{dx^2}G_1(x) = \alpha^2 G_1(x),$$
$$\frac{d^2}{dy^2}G_1(y) = \left(k_u^{\,2} - \alpha^2\right)G_1(y)$$
$$\frac{d^2}{dx^2}G_2(x) = \alpha^2 G_2(x),$$
$$\frac{d^2}{dy^2}G_2(y) = \left(k_h^{\,2} - \alpha^2\right)G_2(y) \qquad (10)$$

where h is the constant. The solutions of the above equations can be given in the form

$$G_1(x) = A\exp(\alpha x) + B\exp(-\alpha x)$$
$$G_1(y) = C\exp\left(\sqrt{k_u^{\,2} - \alpha^2}\,y\right) + D\exp\left(-\sqrt{k_u^{\,2} - \alpha^2}\,y\right)$$
$$G_2(x) = A\exp(\alpha x) + B\exp(-\alpha x)$$
$$G_2(y) = C\exp\left(\sqrt{k_h^{\,2} - \alpha^2}\,y\right) + D\exp\left(-\sqrt{k_h^{\,2} - \alpha^2}\,y\right) \qquad (11)$$

The constants A, B, C, D can be evaluated from a set of the boundary conditions, which can be presented as,

$$G_1(0) = B_0, G_1(0) = 0, \frac{d}{dx}G_1(x)\bigg|_{x=0} = 0, \frac{d}{dy}G_1(y)\bigg|_{y=0} = k_u$$

$$G_2(0)=B_0, G_2(0)=0, \frac{d}{dx}G_2(x)\Big|_{x=0}=0, \frac{d}{dy}G_2(y)\Big|_{y=0}=k_h \quad (12)$$

using above relations and eqs(9,11) we find,

$$G_1(x,y)=\frac{B_0k_u}{\sqrt{k_u^2-\alpha^2}}\cosh(\alpha x)\sinh\left[\sqrt{k_u^2-\alpha^2}\,y\right]$$

$$H_1(x,y)=B_0\cosh(\alpha x)\cosh\left[\sqrt{k_u^2-\alpha^2}\,y\right]$$

$$F_1(x,y)=\frac{B_0\alpha}{\sqrt{k_u^2-\alpha^2}}\sinh(\alpha x)\sinh\left[\sqrt{k_u^2-\alpha^2}\,y\right]$$

$$G_2(x,y)=\frac{B_0k_h}{\sqrt{k_h^2-\alpha^2}}\cosh(\alpha x)\sinh\left[\sqrt{k_h^2-\alpha^2}\,y\right]$$

$$H_2(x,y)=B_0\cosh(\alpha x)\cosh\left[\sqrt{k_h^2-\alpha^2}\,y\right]$$

$$F_2(x,y)=\frac{B_0\alpha}{\sqrt{k_h^2-\alpha^2}}\sinh(\alpha x)\sinh\left[\sqrt{k_h^2-\alpha^2}\,y\right] \quad (13)$$

we are interested in small region around the axis, so we can expand eq(13) keeping contributions upto the second order in the transverse coordinates thus getting,

$$B_x=\left[\frac{B_0\alpha}{\sqrt{k_u^2-\alpha^2}}\sinh(\alpha x)\sinh\left(\sqrt{k_u^2-\alpha^2}.y\right)\{a_1\sin(k_uz)\}\right.$$

$$\left.+\frac{B_0\alpha}{\sqrt{k_h^2-\alpha^2}}\sinh(\alpha x)\sinh\left(\sqrt{k_h^2-\alpha^2}.y\right)\{a_2\sin(k_hz)\}\right]$$

$$B_y=\left[B_0\cosh(\alpha x)\cosh\left(\sqrt{k_u^2-\alpha^2}.y\right)\{a_1\sin(k_uz)\}\right.$$

$$\left.+B_0\cosh(\alpha x)\cosh\left(\sqrt{k_h^2-\alpha^2}.y\right)\{a_2\sin(k_hz)\}\right]$$

$$B_z=\left[\frac{B_0k_u}{\sqrt{k_u^2-\alpha^2}}\cosh(\alpha x)\sinh\left(\sqrt{k_u^2-\alpha^2}.y\right)\{a_1\cos(k_uz)\}\right.$$

$$\left.+\frac{B_0k_h}{\sqrt{k_h^2-\alpha^2}}\cosh(\alpha x)\sinh\left(\sqrt{k_h^2-\alpha^2}.y\right)\{a_2\cos(k_hz)\}\right]$$

(14)

At the lowest order in transverse coordinates these field components read(14),

$$B_x=\frac{B_0}{2}xy\left[k_u^2\delta\{a_1\sin(k_uz)\}+k_h^2\sigma\{a_2\sin(k_hz)\}\right]$$

$$B_y=B_0\left[1+\frac{k_u^2}{4}\left\{\delta x^2+(2-\delta)y^2\right\}\right]a_1\sin(k_uz)$$

$$+B_0\left[1+\frac{k_h^2}{4}\left\{\sigma x^2+(2-\sigma)y^2\right\}\right]a_2\sin(k_hz)$$

$$B_z=B_0y\left[a_1k_u\cos(k_uz)+a_2k_h\cos(k_hz)\right] \quad (15)$$

where δ and σ are the focusing and defocusing parameters in the sextupolar fields. The equation of motion can now be derived using the field equation in (15). We assume that the motion can be decomposed as $x=x_0+x_1$ where x_0 represents the reference trajectory due to the on-axis field specified by Eq(1) and x_1 describe the additional motion around the reference trajectory due to the extra terms in Eq(15) depending on the transverse coordinate. Keeping the contributions at the first order only in x_1 and averaging over one undulator period, we get the following differential equation ruling the additional motion,

$$\frac{d^2x_1}{dt^2}+\Omega_\beta^2x_1=0 \quad (16)$$

where,

$$\Omega_\beta^2=\frac{K_1^2c^2k_u^2}{4\gamma^2}\left[\delta+h^2\sigma\right]$$

The solution of Eq(16) is given by,

$$x_1(t)=x_1(0)\cos(\Omega_\beta t) \quad (17)$$

where $x_1(0)$ represent the off-axis position from the undulator axis and $y_1(0)=0$ is assumed. The z-motion is

$$z(t)=\beta^{**}ct-\frac{cK_1^2}{8\gamma^2\omega_u}\sin(2\omega_ut)-\frac{cK_h^2}{8\gamma^2h\omega_u}\sin(2h\omega_ut)-\frac{cK_1K_h}{2\gamma^2\omega_u(1+h)}\sin\omega_u(1+h)t$$

$$-\frac{cK_1K_h}{2\gamma^2\omega_u(1-h)}\sin\omega_u(1-h)t+\frac{x_1^{\,2}(0)\Omega_\beta}{8c}\sin(2\Omega_\beta t)+\frac{K_1x_1(0)\Omega_\beta}{2\gamma(\omega_u+\Omega_\beta)}\cos(\omega_u+\Omega_\beta)t$$
$$-\frac{K_1x_1(0)\Omega_\beta}{2\gamma(\omega_u-\Omega_\beta)}\cos(\omega_u-\Omega_\beta)t+\frac{K_hx_1(0)\Omega_\beta}{2\gamma(h\omega_u+\Omega_\beta)}\cos(h\omega_u+\Omega_\beta)t-\frac{K_hx_1(0)\Omega_\beta}{2\gamma(h\omega_u-\Omega_\beta)}\cos(h\omega_u-\Omega_\beta)t \tag{18}$$

$$\beta^{**}=1-\frac{1}{2\gamma^2}\left[1+\frac{K_1^{\,2}}{2}+\frac{K_h^{\,2}}{2}+\frac{x_1^{\,2}(0)\Omega_\beta^{\,2}\gamma^2}{2c^2}\right]$$

The spectral properties of the undulator radiation are easily obtained from the Lienard-Wiechert integral [23],

$$\frac{d^2I}{d\omega d\Omega}=\frac{e^2\omega^2}{4\pi^2c}\left|\int_{-\infty}^{\infty}\left\{\hat{n}\times\left(\hat{n}\times\vec{\beta}\right)\right\}\exp\left[i\omega\left(t-\frac{z}{c}\right)\right]dt\right|^2 \tag{19}$$

The integral is evaluated in a region where an effective acceleration exists. Referring to a common configuration of an OK when two undulator section have equal length, we have

$$\int_{-\infty}^{\infty}(....)dt=\int_{0}^{T}(....)dt+\int_{T(1+D)}^{T(2+D)}(....)dt \tag{20}$$

With this, Eq.(19) can be cast in the form,

$$\frac{d^2I}{d\omega d\Omega}=\sum_m\frac{d^2I_m}{d\omega d\Omega}\left\{2+2\cos(\theta_m)\right\} \tag{21}$$

where, $\theta_m=\nu_0(1+D)$

$$\frac{d^2I_m}{d\omega d\Omega}=\frac{e^2\omega^2T^2}{4\pi^2c}\left|F_m(z)\right|^2\left(\frac{\sin(\nu_0/2)}{\nu_0/2}\right)^2$$

where,

$$F_m(z)=\left[\frac{K_1}{2\gamma}\left\{J_{m-1}(0,z_1)+J_{m+1}(0,z_1)\right\}J_n(0,z_2)J_r(0,z_5)\right.$$
$$+\frac{K_h}{2\gamma}\left\{J_{n-1}(0,z_2)+J_{n+1}(0,z_2)\right\}J_m(0,z_1)J_r(0,z_5)$$
$$\left.+\frac{x_1(0)\Omega_\beta}{2ic}\left\{J_{r+1}(0,z_5)-J_{r-1}(0,z_5)\right\}J_m(0,z_1)J_n(0,z_2)\right]$$
$$\times J_p(z_3,0)J_q(z_4,0)J_s(z_6,0)J_u(z_7,0)J_v(z_8,0)J_w(z_9,0) \tag{22}$$

$\frac{d^2I_m}{d\omega d\Omega}$ is the brightness of m^{th} harmonic specify by

$$\nu_0=\frac{\omega}{2\gamma^2}\left(1+\frac{K_1^{\,2}}{2}+\frac{K_h^{\,2}}{2}+\frac{x_1^{\,2}(0)\Omega_\beta^{\,2}\gamma^2}{2c^2}\right)-\eta$$

$$\eta=m\omega_u+n\omega_h+p(\omega_u+\omega_h)+q(\omega_u-\omega_h)+r\Omega_\beta+s(\omega_u+\Omega_\beta)+u(\omega_u-\Omega_\beta)+v(\omega_h+\Omega_\beta)+w(\omega_h-\Omega_\beta)$$

with

$$z_1=-\frac{\omega K_1^{\,2}}{8\gamma^2\omega_u},\ z_2=-\frac{\omega K_h^{\,2}}{8\gamma^2\omega_h},\ z_{3,4}=-\frac{\omega K_1K_h}{2\gamma^2(\omega_u\pm\omega_h)},\ z_5=\frac{\omega x_1^{\,2}(0)\Omega_\beta}{8c^2}$$
$$z_{6,7}=\pm\frac{\omega K_1x_1(0)\Omega_\beta}{2\gamma c(\omega_u\pm\Omega_\beta)},\ z_{8,9}=\pm\frac{\omega K_hx_1(0)\Omega_\beta}{2\gamma c(\omega_h\pm\Omega_\beta)}$$

m,n,p,q,r,s,u,v,w are the harmonic integer numbers for oscillations at

$$\omega_u,\omega_h,\omega_u+\omega_h,\omega_u-\omega_h,\Omega_\beta,\omega_u+\Omega_\beta,\omega_u-\Omega_\beta,\omega_h+\Omega_\beta,\omega_h-\Omega_\beta$$

respectively.

$J_m(0,z_1),J_n(0,z_2),J_p(z_3,0),J_q(z_4,0),J_r(z_5,0),J_s(z_6,0),J_u(z_7,0),J_v(z_8,0),J_w(z_9,0)$

are the generalized Bessel functions. The resonance condition in a free electron laser is provided by $\nu_0 = 0$. This provides the central emission frequency as,

$$\omega_i = \frac{2\gamma^2\eta}{1+\left(K_1^2/2\right)+\left(K_h^2/2\right)+\left(x_1^2(0)\Omega_\beta^2\gamma^2/2c^2\right)} \tag{23}$$

3. Free electron laser gain

To calculate the gain with optical klystron configuration, let us consider a linear polarized electromagnetic wave with

$$\vec{E} = E_0\hat{x}\cos(kz-\omega t+\varphi) \tag{24}$$

The change in energy of the electron is given by,

$$\frac{d\gamma}{d\tau} = \frac{eE_o K_1 L_u}{m_0 c^2\gamma}\cos(\zeta_0+\varphi)J_p(z_3,0)J_q(z_4,0)J_r(0,z_5)J_s(z_6,0)J_u(z_7,0)J_v(z_8,0)J_w(z_9,0)\times \left[\{J_{m-1}(0,z_1)+J_{m+1}(0,z_1)\}J_n(0,z_2)+\Delta\{J_{n-1}(0,z_2)+J_{n+1}(0,z_2)\}J_m(0,z_1)\right] \tag{25}$$

where $\zeta_0 = \eta[(k_1+k_u)\bar{z}-\omega_1 t]$

The pendulum equation is,

$$\frac{d^2\zeta_0}{d\tau^2} = |a|\cos(\zeta_0+\varphi) \tag{26}$$

where the dimensionless field strength is,

$$|a| = \frac{4\pi NeE_0K_1L}{\gamma^2 m_0 c^2}\eta J_p(z_3,0)J_q(z_4,0)J_r(0,z_5)J_s(z_6,0)J_u(z_7,0)J_v(z_8,0)J_w(z_9,0)\times \left[\{J_{m-1}(0,z_1)+J_{m+1}(0,z_1)\}J_n(0,z_2)+\Delta\{J_{n-1}(0,z_2)+J_{n+1}(0,z_2)\}J_m(0,z_1)\right] \tag{27}$$

In its simplest form the wave equation is written as,

$$\frac{da}{d\tau} = -j\left\langle e^{-i\zeta_0}\right\rangle \tag{27}$$

where j is the dimensionless current density given as,

$$j = \frac{4\pi^2 Ne^2K_1^2L^2n_e}{\gamma^3 m_0 c^2}\eta J_p(z_3,0)J_q(z_4,0)J_r(0,z_5)J_s(z_6,0)J_u(z_7,0)J_v(z_8,0)J_w(z_9,0)\times \left[\{J_{m-1}(0,z_1)+J_{m+1}(0,z_1)\}J_n(0,z_2)+\Delta\{J_{n-1}(0,z_2)+J_{n+1}(0,z_2)\}J_m(0,z_1)\right] \tag{28}$$

Eqs.(26 & 28) are simple, but powerful, since they are valid for high and low gain in both weak and strong optical fields. Integrating the modulator section to zeroth order in a_0 we have the solutions from $\tau = 0 \rightarrow 0.5$

$$\nu_<(\tau) = \nu_0 + \ldots\ldots\ldots,\quad \zeta_<(\tau) = \zeta_0 + \nu_0\tau + \ldots\ldots\ldots,$$
$$|a_<(\tau)| = a_0 + \ldots\ldots. \tag{29}$$

The average $<\ldots\ldots.> = \int_0^{2\pi} d\zeta(\ldots\ldots)/2\pi = 0$ when taken over all electron phases with no modulation. To first order in a_0 and j, the solutions for $0 \le \tau < 0.5$ in the modulator are

$$\nu_<(\tau) = \nu_0 + \frac{a_0}{\nu_0}[\sin(\zeta_0+\nu_0\tau)-\sin(\zeta_0)] + \ldots\ldots\ldots\ldots,$$

$$\zeta_<(\tau) = \zeta_0 + \nu_0\tau + \frac{a_0}{\nu_0^2}[-\cos(\zeta_0+\nu_0\tau)+\cos(\zeta_0)-\nu_0\tau\sin(\zeta_0)] + \ldots\ldots,$$

$$|a_<(\tau)| = a_0 + \frac{ja_0}{2\nu_0^3}[2-2\cos(\nu_0\tau)-\nu_0\tau\sin(\nu_0\tau)] + \ldots\ldots.., \tag{30}$$

The dispersion section interaction is at $\tau = 0.5$. The solutions for electrons entering the dispersion section are with $\tau = 0.5$, so immediately after we have

$$\nu(0.5) = \nu_<(0.5),$$
$$\zeta(0.5) = \zeta_<(0.5) + \nu_<(0.5)D,$$
$$|a(0.5)| = |a_<(0.5)|, \tag{31}$$

For $\tau > 0.5$, we must continue integrating (16 & 18) with (22) as the initial conditions. To zeroth order in fields, the solutions in the radiator section for $\tau > 0.5$ are

$$\nu_> = \nu_0 + \ldots\ldots\ldots,$$
$$\zeta_> = \zeta_0 + \nu_0\tau + \nu_0 D + \ldots..,$$
$$|a_>| = a_0 + \ldots\ldots, \tag{32}$$

The first order changes in the electron velocity and phase are

$$\nu_{>}(\tau)=\nu_0+\frac{a_0}{\nu_0}\left[\sin(\zeta_0+\nu_0/2)-\sin(\zeta_0)+\sin(\zeta_0+\nu_0\tau+\nu_0 D)-\sin(\zeta_0+\nu_0/2+\nu_0 D)\right]+.....,$$

$$\begin{aligned}\zeta_{>}(\tau)&=\zeta_0+\nu_0\tau+\nu_0 D+\frac{a_0}{\nu_0^{2}}\big[-\cos(\zeta_0+\nu_0/2)+\cos(\zeta_0)-(\nu_0/2)\sin(\zeta_0)\\&+\nu_0(\tau+D-1/2)[\sin(\zeta_0+\nu_0/2)-\sin(\zeta_0)]-\cos(\zeta_0+\nu_0\tau+\nu_0 D)\\&+\cos(\zeta_0+\nu_0/2+\nu_0 D)-\nu_0(\tau-1/2)\sin(\zeta_0+\nu_0/2+\nu_0 D)\big]+.......,\end{aligned} \quad (33)$$

The optical field is calculated in the same way using the field at $\tau=0.5$ as initial conditions.

$$\begin{aligned}|a_{>}(\tau)|&=a_0-\frac{ja_0}{2\nu_0^{3}}\big[-2+2\cos(\nu_0/2)+(\nu_0/2)\sin(\nu_0/2)+2\cos(\nu_0(\tau-1/2))\\&+\nu_0(\tau-1/2)\sin(\nu_0(\tau-1/2))-2+2\cos(\nu_0(\tau+D))+\nu_0(\tau+D)\sin(\nu_0(\tau+D))\\&-2\cos(\nu_0(D+1/2))-\nu_0(D+1/2)\sin(\nu_0(D+1/2))-2\cos(\nu_0(\tau+D-1/2))\\&-\nu_0(\tau+D-1/2)\sin(\nu_0\tau+\nu_0 D-\nu_0/2)+2\cos(\nu_0 D)+\nu_0 D\sin(\nu_0 D)\big]\end{aligned}$$

(34)

A compact way of writing the lengthy result is

$$|a_{>}(\tau)|=a_0-\frac{ja_0}{2\nu_0^{3}}[2\cos(x)+\sin(x)]\left[\left[\Big|_{0}^{\nu_0/2}+\Big|_{0}^{\nu_0(\tau-1/2)}+\Big|_{\nu_0(D+1/2)}^{\nu_0(\tau+D)}-\Big|_{\nu_0 D}^{\nu_0(\tau+D-1/2)}\right]\right] \quad (35)$$

where $f(x)\Big|_a^b=f(b)-f(a)$. The gain is given by $G=2(|a(\tau)|-a_0)/a_0$

The final gain at $\tau=1$

$$|G(\nu_0)|=-\frac{j}{\nu_0^{3}}[2\cos(x)+\sin(x)]\left[\left[2\Big|_{0}^{\nu_0/2}+\Big|_{\nu_0(D+1/2)}^{\nu_0(D+1)}-\Big|_{\nu_0 D}^{\nu_0(D+1/2)}\right]\right] \quad (36)$$

4. Results and discussion

In this paper we have examined betatron oscillation effects in a harmonic undulator optical klystron scheme. The betatron oscillations in the system arises due to sextupolar field contributions and introduces significant modifications to the undulator radiation through the modification of the central emission frequency and generation of on-axis even harmonics. The inclusion of the betatron contributions gives a clear idea of how the off-axis motion induces not only a shift of the central emission frequency but also a variation in the intensity radiated at a fixed harmonic. For the case of a regular planar undulator the intensity expression and central emission frequency is recovered with substitution of $K_h=0$ in Eq.(22).

$$F_m=\left[\frac{K_1}{2\gamma}\{J_{m-1}(0,z_1)+J_{m+1}(0,z_1)\}J_r(0,z_5)+\frac{x_1(0)\Omega_\beta}{2ic}\{J_{r+1}(0,z_5)-J_{r-1}(0,z_5)\}J_m(0,z_1)\right]J_s(z_6,0)J_u(z_7,0)$$

The central emission frequency is read from the resonance condition and is provided by,

$$\omega=\frac{2\gamma^2\left[m\omega_u+r\Omega_\beta+s(\omega_u+\Omega_\beta)+u(\omega_u-\Omega_\beta)\right]}{1+\frac{K_1^{2}}{2}+\frac{x_1^{2}(0)\Omega_\beta^{2}\gamma^{2}}{2c^{2}}}$$

The physical contents of both the expressions are that with the inclusion of betatron oscillations the on-axis emission pattern is complicated since entirely new contributions appear containing Bessel functions of the order $J_r(0,z_5), J_{r\pm1}(0,z_5), J_s(z_6,0)J_u(z_7,0)$.

The arise of these new terms are due to the fact that the transverse electron motion has oscillatory component at the frequency Ω_β and to their mutual interference i.e., $\omega_u \pm \Omega_\beta$ These effects are illustrated in Fig.[0] for odd on-axis harmonics. The intensity and gain with betatron oscillations are reduced for various off-axis entry of the electron. The reduction in intensity and gain are much substantial in comparison to the fundamental. Higher harmonics shows greater sensitivity in comparison to the fundamental.

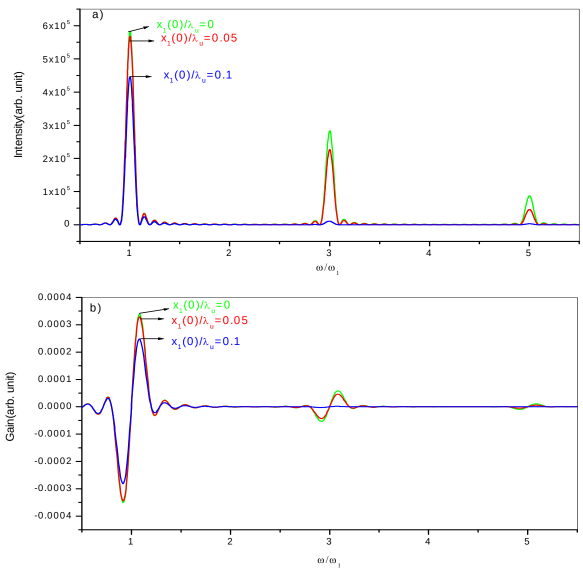

Fig. 0 Intensity and gain curves for a regular planar undulator.

The optical klystron free electron laser with conventional linearly polarized undulator sections, emit at odd harmonic on-axis [Fig.1]. Harmonic undulator schemes are characterized by additional field components polarized either parallel or perpendicular to the standard undulator. In our scheme of study we have adopted a harmonic field which is parallel polarized and whose amplitude is define as the $K_h, K_h = K_1\Delta$. In Fig.[2-3] we analyze the harmonic klystron undulator radiation with $h=2$. The intensity are plotted with harmonic field amplitude ratio, $\Delta = a_2/ha_1$. The electron longitudinal motion shows modulation at $h\omega_u$ as well as at sum, difference frequency $\omega_u \pm h\omega_u$. This has the implication that the system now emits at both even $(h=2,4...)$ and odd harmonics $(h=3,5...)$. In Fig[2] we plot first four harmonics for $h=2$ with $\Delta = 0.25, 0.5, 1$ the intensities at odd harmonic fall whereas the intensities for the even harmonic increases.

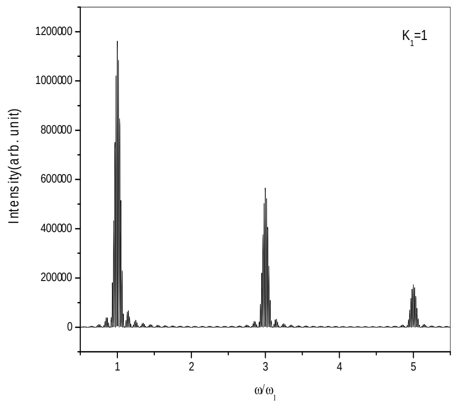

Fig.[1]

Fig. 1 Odd harmonics for a Klystron Undulator radiation.

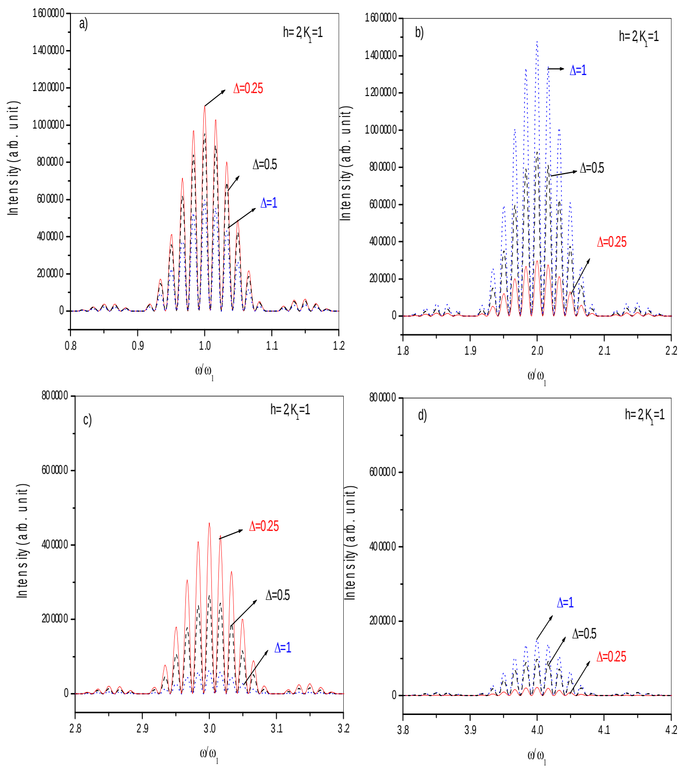

Fig.[2]

Fig. 2 Odd & even harmonics of a harmonic-Klystron Undulator radiation for $h=2$.

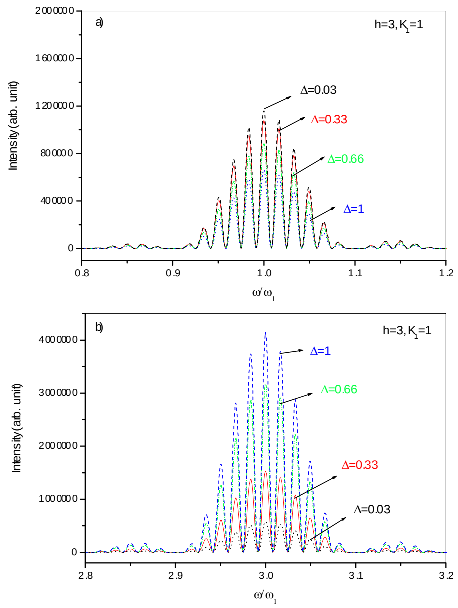

Fig.[3]

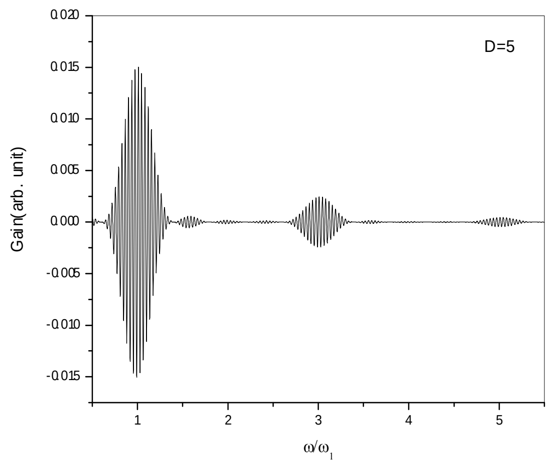

Fig.[4]

Fig. 4 Klystron FEL gain at odd harmonics.

Fig. 3 Odd harmonics of a Klystron Undulator radiation for $h=3$.

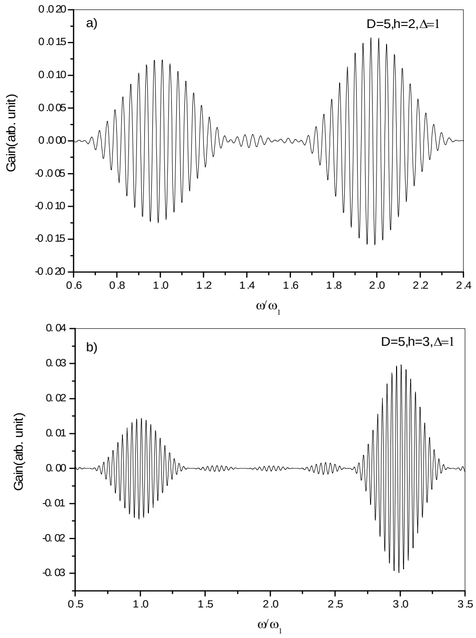

Fig.[5]

Fig. 5 Harmonic klystron FEL gain for a) $h = 2$ *, b)* $h = 3$.

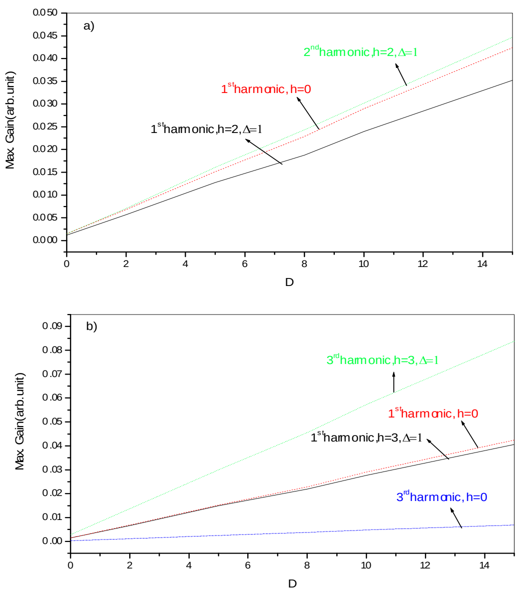

Fig.[6]

Fig. 6 Harmonic klystron FEL gain versus D *for both* 2^{nd} $(h = 2)$ *and third harmonic* $(h = 3)$.

Fig [4], calculates the gain for the standard klystron-FEL with linear undulator fields. The gain at the fundamental is higher than the higher harmonics. In Fig[5], we plot the gain curves for the harmonic klystron FEL for $h = 2$ and $h = 3$. The harmonic amplitude ratio is kept at $\Delta = 1$. The gain at the higher harmonic is higher than the fundamental. In a standard klystron FEL, the gain increases linearly with 'D' where D defines the strength of the dispersive section. In a harmonic undulator OK free electron laser the enhancement in the gain for the second & third on-axis harmonic is quite substantial for higher values of D as shown in Fig.[6]. This provides an interesting possibility to extract higher gain at higher harmonics from a klystron free electron laser.

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