A topological index of a graph $G$ is a numeric quantity related to $G$ which is describe molecular graph $G$. A dendrimer is an artificially manufactured or synthesized molecule built up from branched units called monomers. In this paper the Szeged Index of an infinite class of nanostar dendrimers are computed.

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1. Introduction

Let $G$ be a simple molecular graph without directed and multiple edges and without loops, the vertex and edge-sets of which are represented by $V(G)$ and $E(G)$, respectively. In such a graph, vertices represent atoms and edges represent bonds. The graph $G$ is said to be connected if for every vertices $x$ and $y$ in $V(G)$ there exists a path connecting $x$ and $y$. The distance $d(u,v)$ between vertices $u$ and $v$ of a connected graph $G$ is the number of edges in a minimum path from $u$ to $v$. A topological index is a real number related to a molecular graph, which is a graph invariant. There are several topological indices already defined. The Wiener index $W$ is the first topological index that was introduced in 1947 by Harold Wiener.\(^1\) It is defined as the sum of distances between all pairs of vertices in the graph under consideration. The Szeged index is another topological index which is acquaintance with Ivan Gutman.\(^2,4\) To define the Szeged index of a graph $G$, we assume that $e = uv$ is an edge connecting the vertices $u$ and $v$. Suppose $n_u(e)$ is the number of vertices of $G$ lying closer to $u$ and $n_v(e)$ is the number of vertices of $G$ lying closer to $v$. Then the Szeged index of the graph $G$ is defined as $Sz(G) = \sum_{e=uv \in E(G)}[n_u(e)n_v(e)]$. Notice that vertices equidistance from $u$ and $v$ are not taken into account.

Diudea\(^5-10\) was the first scientist investigated the mathematical properties of nanostructures. He and his team considered too many nanostructures into account by computing their topological indices and counting polynomials. The first author of this paper continued the works of Diudea by computing the topological indices of some new type of nanostructures.\(^11-16\)

Throughout this paper $G[n] = NSC_5C_6[n]$ denotes the nanostar dendrimer of Figure 1. We encourage the reader to consult papers\(^17,18\) and book\(^19\) for further study on this topic. Our notation is standard and taken mainly from the standard book of graph theory.

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2. Main Results and discussion

The aim of this section is to compute the Szeged index of nanostar dendrimers NSC₃₆[n], depicted in Figures 1 and 2. This nanostar dendrimer has a core depicted in Figure 1. Using an inductive argument, one can show that |V(NSC₃₆[n])| = 9.2ⁿ⁺² − 44.

We begin by computing values of ⁵_u_i(v_i) and ⁵_v_i(v_i) for an arbitrary edge e = u_i,v_i of the hexagon N_i, 1 ≤ i ≤ n. Again apply an inductive argument to prove ⁵_u_i(v_i) = |V(G[n])| − ⁵_v_i(v_i) and ⁵_v_i(v_i) = 9⋅2ⁿ⁻𝑖⁺２ − 22. Values of other edges is computed in Table 1.

Consider the edges e = u_i,v_i of pentagons A_i, 1 ≤ i ≤ n. Then ⁵_u_i(v_i) = |V(G[n])| − ⁵_v_i(v_i) = 9⋅2ⁿ⁻𝑖⁺１ − 14 and ⁵_v_i(v_i) = 9⋅2ⁿ⁻𝑖⁺１ − 14. Suppose e = u_i,v_i is an edge of type z₂z₃ or z₄z₅ of pentagons A_i, 1 ≤ i ≤ n. Then ⁵_u_i(v_i) = |V(G[n])| − ⁵_v_i(v_i) − 1 and ⁵_v_i(v_i) = 18⋅2ⁿ⁻𝑖⁺１ − 30. To complete the investigation of of pentagons, we must consider the edge e = z₃z₄. A similar calculations show that ⁵_u_i(v_i) = ⁵_v_i(v_i) = 9⋅2ⁿ − 14.
Table 1. The Values of $n_d(e)$ of the Nanostar Dendrimer $G[n]$.

<table>
<thead>
<tr>
<th>Edges</th>
<th>The Values of $n_d(e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e = uv = x_iy_j$</td>
<td>$n_d(e) = 18 \cdot 2^{n+i+j} - 25$</td>
</tr>
<tr>
<td>$e = uv = m_in_i$</td>
<td>$n_d(e) = 9 \cdot 2^{n+i+2} - 19$</td>
</tr>
<tr>
<td>$e = uv = st$</td>
<td>$n_d(e) = 1$</td>
</tr>
<tr>
<td>$e = uv = o_ih_i$</td>
<td>$n_d(e) = 18 \cdot 2^{n+i+1} - 27$</td>
</tr>
<tr>
<td>$e = uv = d_id_d_2$ or $d_id_d_4$</td>
<td>$n_d(e) = 9 \cdot 2^{n+i+1} - 16$</td>
</tr>
<tr>
<td>$e = uv = g_ig_2$</td>
<td>$n_d(e) = 9 \cdot 2^{n+i+1} - 17$</td>
</tr>
</tbody>
</table>

By computing the number of edges and some simple calculations by MAPLE, we can prove the following theorem:

**Theorem.** The Szeged index of the nanostar dendrimer $NSC_3C_6[n]$ is computed as $Sz(NSC_3C_6[n]) = -15846 + 41828.2n - 21636.4n - 4320.8n - 4068.2n + 11664.n.4n$.

**References**