

## MATHEMATICAL MODELING FOR THE SIMULATION OF HEAVY METAL IONS ADSORPTION BY SINGLE WALL CARBON NANOTUBES (SWCNTs) BASED ON COMPUTATIONAL CALCULATION

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We performed this work to calculate the carbon nanotube parameters for the  $M^{+2}@SWCNT$  model such as incapsulated  $Hg^{+2}$  and  $Pb^{+2}$  metal ions with nanotubes. With using this mathematic modeling for other  $M^{+2}@SWCNT$  models, based on total interaction energy between metal ions and carbon nanotube and this mathematical modeling, we can design a new approach for separation of mixture metal ions based on nanotechnology.

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*Keywords: Mathematical modeling, nanotechnology, nanotube,  $M^{+2}@SWCNT$ .*

### 1. Introduction

After discovery of carbon nanotubes (CNTs) [1], numerous works have been devoted to study the properties and applications of this fascinating novel material [2]. The applications of CNTs are widely ranged from nano-biotechnology to nano-mathematic, e.g., [3,4]. However, based on nanotubes properties exhibiting absorption or storageing behavior depending upon the tubular diameter and chirality. Therefore, considerable efforts have been raised to synthesize CNTs with properties independent of tubular diameter and chirality [5].

Among them, the group of heavy metals and especially  $Hg^{+2}$  and  $Pb^{+2}$ , which always exhibit toxic behavior, are considered as proper alternates of other heavy metals [6].

The stable one-dimensional structure of CNT was either theoretically recognized or experimentally synthesized [1,7]. Previous studies indicated that CNTs are very important in the gas storage and electrochemical determination of the metal ions or molecules but the mathematical study for the preparation of the mathematical modeling has not been reported [8, 9]. This work studies the existence of mathematical modeling for separation of the metal ions based on the Gaussian 98 software. Density functional theory (DFT) calculations on two considered structures of CNTs and  $CNTs@M^{+2}$  are performed employing BLYP methods and b3lyp/6-31+g\* opt# standard basis set. The optimized geometry is exhibited in figure 1- 6.

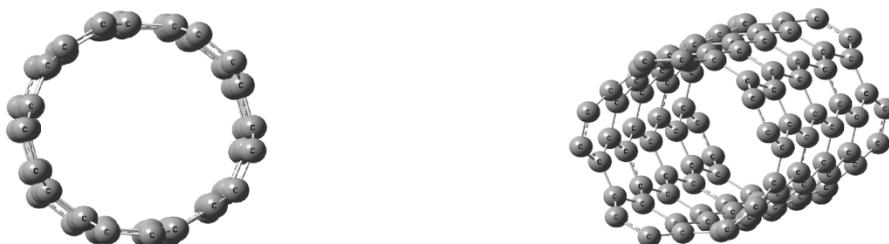
$$E_{tot} = [E_{M@CNT} - (E_M + E_{CNT})] * 627.529 \quad \text{k cal/mol}$$

**E.q. 1:** Mathematic equation for total energy of heavy metal absorption.

Since optimal ground state energy is very useful elements to study the free structure and then comparison with optimized energy of  $M^{+2}@CNTs$ , calculation of RHF (Restricted Hartree-Fock) with then dot to 627.529 factor is used in this paper as a energy level based on k cal/mol Eq. 1.



*Fig. 1. The side and end view of geometry optimized (4,0) structure.*



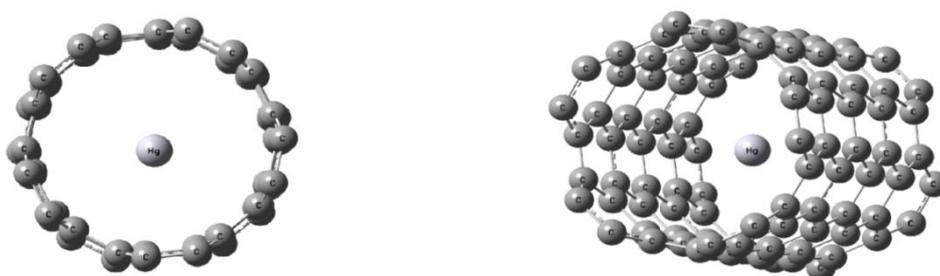
*Fig. 2. The side and end view of geometry optimized (5,0) structure.*



*Fig. 3. The side and end view of geometry optimized (5,5) structure.*



*Fig. 4. The side and end view of geometry optimized  $Hg^{+2}@ (4,0)$  CNT structure.*



*Fig. 5. The side and end view of geometry optimized  $Hg^{+2}@ (5,0)$  CNT structure.*

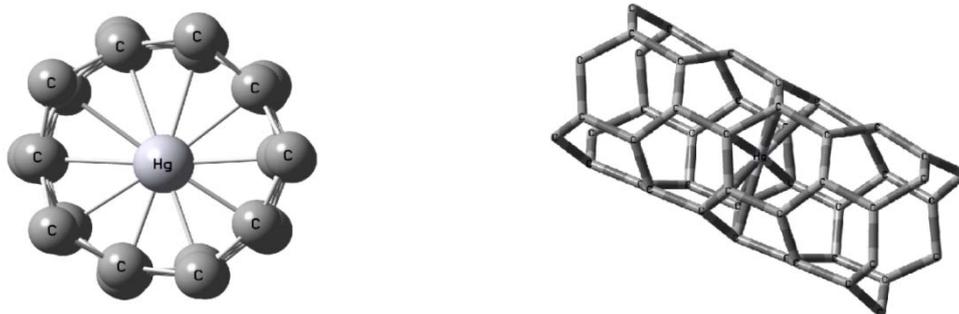


Fig. 6. The side and end view of geometry optimized  $Hg^{+2}@ (5,5)CNT$  structure.

In the present study as a efficient mathematical modeling, a theoretical work has been carried out on a single-walled and endohedrally  $M^{+2}@SWCNT$  such as (4,0), (5,0) and (5,5),  $Hg^{+2}@C50H10$ ,  $Hg^{+2}@C100H20$  and  $Hg^{+2}@C50H10$ , for heavy metal separation, respectively, selected.

## 2. Computational procedure

In this study, two representative models of the SW zigzag and armchair CNTs are considered in the Gaussian calculations. The first model is the (4,0) SW-CNT consisting of 50 C and  $Hg^{+2}$  or  $Pb^{+2}$  atoms where the two ends of the tube are capped by 10 H atoms (see Fig. 1 and 4). The second model is the (5,0) SW-CNT consisting of 100 C and  $Hg^{+2}$  or  $Pb^{+2}$  atoms where the two ends of the tube are capped by 20 H atoms (see Fig. 2 and 5). The third model is the (5,5) SW-CNT consisting of 50 C and  $Hg^{+2}$  or  $Pb^{+2}$  atoms where the two ends of the tube are capped by 10 H atoms (see Fig. 3 and 6). The quantum calculations were performed on the considered models using the Gaussian 98 [10] package of program. The quantum calculations were performed on the geometrical optimized models by the B3LYP method and the # b3lyp/6-31+G\* standard basis set to evaluate the Hartree -Fock energy and other parameters (see Tables 1 and 2).

## 3. Results and discussion

At the first step of this study, each of the considered zigzag and armchair representative models of SW-CNT was allowed to fully relax during the geometrical optimization at the level of the B3LYP DFT method and the # b3lyp/6-31+G\* standard basis set. Tables 1 present the optimized geometries of the models Figs. 4, 5 and 6. And, table 2 present the optimized geometries of the models  $Pb^{+2}$  in the hole of Figs. 1, 2 and 3.

Table 1: The example mathematical calculation energy for  $SWCNT@Hg^{+2}$

SWCNT	Etot kcal/mol
Armchair (4,0)	<b>-120.54</b>
Armchair (5,0)	-112.58
Chiral (5,5)	-110.96

Table 2: The example mathematical calculation energy for  $SWCNT@Pb^{+2}$

SWCNT	Etot kcal/mol
Armchair (4,0)	<b>-115.47</b>
Armchair (5,0)	-110.89
Chiral (5,5)	-108.11

The results shown in table 1 and 2 indicates that interestingly the  $\text{Hg}^{+2}$  and  $\text{Pb}^{+2}$  can absorb inter hole of nanotubes, also the diameters of armchair (4,0) structures have better interaction with metal ions because those have less exothermic energy.

Based on the table 1 and comparison with 2, total energy for  $\text{Hg}^{+2}@(\text{4,0})\text{SWCNT}$  is more exothermic than  $\text{Pb}^{+2}@(\text{4,0})\text{SWCNT}$ , however, armchair (4,0) SWCNT could apply for separation of the mixture ions of  $\text{Hg}^{+2}$  and  $\text{Pb}^{+2}$ . With using this mathematic modeling for other heavy metals, based on total interaction energy between metal ions and carbon nanotube and this mathematical modeling, we can design a new approach for separation of industrial waste mixture metal ions and others.

Table 4: The summarized example mathematical calculation parameters in the (4,0) SWCNT@ $\text{Hg}^{+2}$

Mathematical simulation for $\text{Hg}^{+2}@(\text{4,0})\text{SWCNT}$					
(Summarized)					
*****					
Gaussian 03: IA32W-G03RevD.01 13-Oct-2005					
26-Oct-2008					
*****					
%chk=2					
%mem=500MB					
-----					
# b3lyp/6-31+G* opt extrabasis					
-----					
1/38=1/1;					
2/17=6,18=5,40=1/2;					
3/6=3,10=1,11=2,16=1,25=1,30=1,74=-5/1,2,3;					
4//1;					
5/5=2,32=1,38=5/2;					
6/7=2,8=2,9=2,10=2,28=1/1;					
99/5=1,9=1/99;					
-----					
Title Card Required					
-----					
Symbolic Z-matrix:					
Charge = 0 Multiplicity = 1					
C	0.	0.	0.		
H	1.01627	0.	0.		
C	-2.04819	1.42995	0.		
				.	
				.	
				.	
C	-9.15284	-0.27869	-1.96889		
Hg	-4.29226	1.36554	-2.28034		
Stoichiometry C50H10Hg					
Framework group C1[X(C50H10Hg)]					
Deg. of freedom 177					
Full point group C1					
Largest Abelian subgroup C1 NOp 1					
Largest concise Abelian subgroup C1 NOP 1					
Standard orientation:					
-----					
Center	Atomic	Atomic	Coordinates (Angstroms)		
Number	Number	Type	X	Y	Z
-----					

```

1      6      0      4.613267 -0.183668 -2.160334
2      1      0      5.622853 -0.172540 -2.044471
      .
      .
      .
60     6      0     -4.661799  1.052104 -1.742304
61     80     0      0.045054 -0.080345 -0.013492
-----
Rotational constants (GHZ):  0.1918900  0.0696625  0.0696266
Standard basis: 6-31+G* (5D, 7F)
There are 278 symmetry adapted basis functions of A symmetry.
Integral buffers will be 262144 words long.
Raffenetti 2 integral format.
Two-electron integral symmetry is turned on.
278 basis functions, 810 primitive gaussians, 280 cartesian basis functions
195 alpha electrons 195 beta electrons
nuclear repulsion energy 10285.8090264181 Hartrees.
      .
      .
      .
NAtoms= 61 NActive= 61 NUniq= 61 SFac= 7.50D-01 NAtFMM= 80
NAOKFM=F Big=T
One-electron integrals computed using PRISM.
NBasis= 278 RedAO= T NBF= 278
NBsUse= 278 1.00D-06 NBFU= 278
Defaulting to unpruned grid for atomic number 80.
Harris functional with IExCor= 402 diagonalized for initial guess.
ExpMin= 4.26D-02 ExpMax= 7.16D+01 ExpMxC= 7.16D+01 IAcc=2 IRadAn=
4 AccDes= 0.00D+00
HarFok: IExCor= 402 AccDes= 0.00D+00 IRadAn= 4 IDoV=1
ScaDFX= 1.000000 1.000000 1.000000 1.000000
Defaulting to unpruned grid for atomic number 80.
Initial guess orbital symmetries:
Occupied (A) (A)
      .
      .
      .
(A) (A)
Mulliken atomic charges:
1
1 C -0.118403
2 H 0.097701
      .
      .
      .
60 C -0.036327
61 Hg 47.817726
Sum of Mulliken charges= 0.00000
Electronic spatial extent (au): <R**2>= 17791.9414
Charge= 0.0000 electrons
Dipole moment (field-independent basis, Debye):
X= -8.2051 Y= -5.6470 Z= -1.0745 Tot= 10.0183
Quadrupole moment (field-independent basis, Debye-Ang):
XX= -739.0291 YY= -854.0948 ZZ= -855.5219
XY= -0.4453 XZ= -0.2133 YZ= -0.1567

```

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Traceless Quadrupole moment (field-independent basis, Debye-Ang):
  XX= 77.1862 YY= -37.8795 ZZ= -39.3067
  XY= -0.4453 XZ= -0.2133 YZ= -0.1567
Octapole moment (field-independent basis, Debye-Ang**2):
  XXX= -370.2324 YYY= 52.8066 ZZZ= 10.0890 XYY= -69.9395
  XXY= 2.8571 XXZ= -0.3846 XZZ= -74.1530 YZZ= 19.1769
  YYZ= 0.8164 XYZ= 1.4202
Hexadecapole moment (field-independent basis, Debye-Ang**3):
  XXXX=-15348.4471 YYYY= -5867.5167 ZZZZ= -5850.6177 XXXY= 28.0668
  XXXZ= -2.4100 YYYYX= 1.8558 YYYZ= -12.5603 ZZZX= -5.2509
  ZZZY= 8.5331 XXYY= -4104.8612 XXZZ= -4085.4512 YYZZ= -1943.9803
  XXYZ= -7.2043 YYXZ= 3.6073 ZZXY= -5.8527
.
.
.
evD.01|State=1-A|HF=-3463.6644426|RMSD=9.176e-005|Thermal=0.|Dipole=-3
.279406,1.7797181,1.2702808|PG=C01 [X(C50H10Hg1)]||@

```

#### 4. Conclusion

We performed this work to calculate the carbon nanotube parameters in the representative zigzag and armchair models of SWCNT for the  $M^{+2}$ @SWCNT model such as encapsulated  $Hg^{+2}$  and  $Pb^{+2}$  metal ions with nanotube. To this end, the geometry optimization and ground state energy calculations were performed on three proper models of H-capped (4,0), (5,0) and (5,5) SWCNTs. From the results, some trends were obtained. First, the results shown in table 1 and 2 indicates that the  $Hg^{+2}$  and  $Pb^{+2}$  can absorb inter hole of nanotubes, also the diameters of armchair (4,0) structures have better interaction with metal ions. Second, based on the table 1 and comparison with 2, total energy for  $Hg^{+2}$ @(4,0)SWCNT is more exothermic than  $Pb^{+2}$ @SWCNT. Third, armchair (4,0) SWCNT could apply for separation of the mixture ions of  $Hg^{+2}$  and  $Pb^{+2}$ . Fourth, finally with using this mathematic modeling for other  $M^{+2}$ @SWCNT models, based on total interaction energy between metal ions and carbon nanotube and this mathematical modeling, we can design a new approach for separation of mixture metal ions.

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