Digest Journal of Nanomaterials and Biostructures Vol. 6, No 3, July - September 2011, p. 1199-1205

THE K-CONNECTIVITY INDEX OF AN INfiNITE CLASS OFDENDRIMER NANOSTARS

WEIQING WANG, XINMIN HOU*, WENJIE NING
Department of Mathematics, University of Science and Technology of China, Hefei 230026, P.R.China

The k-connectivity index ${}^k\chi(G)$ of a molecular graph G is the sum of the weights $(d_{v1}d_{v2}\cdots d_{vk+1})^{-1/2}$, where $v_1v_2\cdots v_{k+1}$runs over all paths of length k in G and d_{vi} denotes the degree of vertex vi. In this paper, we give the explicitly formula of the k-connectivity index of a finite class of dendrimer s, which generalized Ahmadiand Sadeghimehr's result [Second-order connectivity index of an infinite class of dendrimer nanostars, Dig. J. Nanomater Bios., 2009].

(Received June 14, 2011; accepted August 23, 2011)

Keywords: Dendrimer, k-Connectivity Index

1 Introduction

A dendrimer is generally described as a macromolecule, which is characterized by its highly branched 3D structure that provides a high degree of surface functionality and versatility. It is constructed through a set of repeating chemical synthesis procedures that build up from the molecular level to the nanoscale region under the condition that is easily performed in a standard organic chemistry laboratory.

Dendrimers have often been referred to as the"Polymers of the 21st century". Dendrimer chemistry was first introduced in 1978 by Buhleier, Wehner, and Vogtle [3], and in 1985, Tomalia et al [14] synthesized the first family of dendrimers. In 1990, a convergent synthetic approach was introduced by Hawker and Frechet [4]. Dendrimer popularity then greatly increased, resulting in a large number of scientific papers and patents.

Let G be a simple connected graph of order n. In 1975, Randic [10] introduced the connectivity index (now called also Randic index) as ${}^1\chi(G)=\sum_{uv}\frac{1}{\sqrt{d_ud_v}}$,where uv runs over all edges of G. This index has been successfully related to chemical properties, namely if G is the molecular graph of an alkane, then ${}^1\chi(G)$ has a strong correlation with the boiling point and the stability of the compound [8, 9, 12].

The k-connectivity index of an organic molecule whose molecule graph is G is defined As

$$ {}^1\chi(G)=\sum_{v_1v_2\cdots v_{k+1}}(d_{v1}d_{v2}\dots d_{vk+1})^{-\frac{1}{2}} $$

where $v_1v_2\cdots v_{k+1}$ runs over all paths of length k in G and d_{vi} denotes the degree of vertex v_i.
The higher connectivity indices are of great interest in molecular graph theory, one can refer [6] and [13] for more details, and some of their mathematical properties have been reported in [2, 5, 7, 11].

In [1], Ahmadi and Sadeghimehr determined the 2-connectivity index of an infinite class of dendrimer nanostars. In this paper, we give the exact value of the k-connectivity index of such dendrimers for a nonnegative integer k, which generalize Ahmadi and Sadeghimehr's result.

* Corresponding author email: xmhou@ustc.edu.cn

2. Main results

Let D[n] denote a type of dendrimer with n growth stages, D[2], D[3] and D[5] are shown in Fig.1. The dendrimer D[n] can be constructed recursively: set D[1] := $K_{1,4}$ the star with four leaves (vertices of degree one), and D[n + 1] is obtained from D[n] by adding two new independent vertices adjacent to each of the leaves of D[n]. The unique vertex of degree four in D[n] is called the center of D[n].

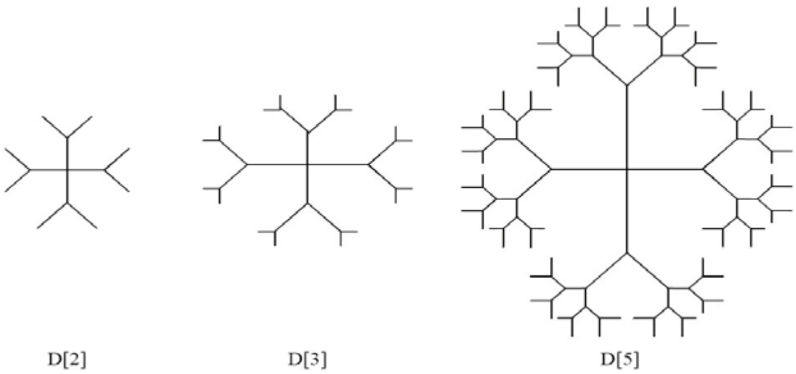

Fig. 1

For a given positive integer k, let $P^{(n)}_{i_1 i_2 \cdots i_{k+1}}$ denote the number of paths composed by k+1 consecutive vertices of degree $i_1, i_2, \cdots, i_{k+1}$, respectively in D[n]. Since D[n] is undirected, $P^{(n)}_{i_1 i_2 \cdots i_{k+1}} = P^{(n)}_{i_{k+1} i_k \cdots i_1}$.
We compute $P_{i_1 i_2 \cdots i_{k+1}}$ according to the choices of $i_1 i_2 \cdots i_{k+1}$.

I. $i_1 i_2 \cdots i_{k+1} = 1\underbrace{3\cdots3}_{k-1}1$. Such paths exist if and only if k is even and 2 ≤ k ≤ 2n−2.

Such a path must start from a leaf, then k/2 steps toward to the center and k/2 steps away from the center. There are $4 \cdot 2^{n-1}$ ways to choose an end of such a path (as there are $4 \cdot 2^{n-1}$ leaves). Then the following k/2 consecutive vertices are uniquely determined (toward the center). Since the next step must toward the reverse direction, this vertex again is determined uniquely. For each of the remaining k−1 vertices, there are two choices, so there are totally $2^{k/2-1}$ ways to choose them. By the symmetry, each path is calculated twice. Hence

$$P^{(n)}_{13\cdots31} = 4\cdot 2^{n-1}\cdot 2^{\frac{k}{2}-1}\cdot\frac{1}{2} = 2^{\frac{k}{2}+n-1}, 2 \le k \le 2n-2$$

II. $i_1 i_2 \cdots i_{k+1} = 1\underbrace{3\cdots3}_{k-1}4$. Such paths exist if and only if k = n.

Such a path is uniquely determined by the end of degree one. So

$$P^{(n)}_{13\cdots34} = 4\cdot 2^{n-1} = 2^{n+1}, k = n$$

III. $i_1 i_2 \cdots i_{k+1} = \underbrace{3\cdots3}_{k}4$. Such paths exist if and only if 1 ≤ k ≤ n − 1.

By the recursion of D[n], such a path can be seen as a path in D[k] of type 13···34. Hence, by II,

$$P^{(n)}_{33\cdots34} = P^{(k)}_{13\cdots34} = 2^{k+1}, 1 \le k \le n-1$$

IV. $i_1 i_2 \cdots i_{k+1} = 1\underbrace{3\cdots3}_{k_1}4\underbrace{3\cdots3}_{k_2}1$ ($k_1 + k_2 = k-2$). Such paths exist if and only if $k_1 = k_2 = n-1$ and k = 2n.

Such a path is composed by two symmetric segments of length k_2 = n, each segment can be considered as a path from the center to a vertex of degree one. There are $\binom{4}{2}$ choices for the first vertex adjacent to the center of the two segments. For each of the remaining k−2 vertices in the two segments, there are two choices to take them. Hence

$$P^{(n)}_{13\cdots4\cdots31} = \binom{4}{2}\cdot 2^{k-2} = 3\cdot 2^{k-1}, k = 2n$$

V. $i_1 i_2 \cdots i_{k+1} = 1\underbrace{3\cdots3}_{k_1}4\underbrace{3\cdots3}_{k_2}$ ($k_2 \ge 1$, $k_1 + k_2 = k-1$). Such paths exist if and only if $k_1 = n-1$, $k_2 = k-n$, $n+1 \le k \le 2n-1$.

Such a path is composed by two segments of length $k_1 + 1$ and k_2, respectively, each of which starts from the center. The difference between the case from case IV is that the two segments are not symmetric. So, by a similar reason as in IV,

$$P^{(n)}{}_{13\cdots4\cdots3} = 4\cdot 3\cdot 2^{k-2} = 3\cdot 2^{k}, n+1 \le k \le 2n-1$$

VI. $$i_1 i_2 \cdots i_{k+1} = 1\underbrace{3\cdots3}_{k_1}4\underbrace{3\cdots3}_{k_2}$$

(VI.1) k is even and $1 \le k \le n-1$.

Such a path must start from a vertex of degree one to a vertex of degree three with $k/2+1$ steps toward to the center, then $k/2-1$ steps toward or away from the center. There are $4\cdot2^{n-1}$ choices for the vertex of degree one, and the first $k/2+2$ vertices are uniquely determined once the starting vertex of degree one has been chosen. For each of the remaining $k/2-1$ vertices, there are two choices. So

$$P^{(n)}{}_{13\cdots33} = 4\cdot 2^{n-1}\cdot 2^{\frac{k}{2}-1} = 2^{\frac{k}{2}+n}$$

(VI.2) k is even and $n \le k \le 2n-4$.

Such a path must start from a vertex of degree one to a vertex of degree three with i steps with $k/2+1 \le i \le n-1$ toward the center, then $k-i$ steps away from the center. There are $4\cdot2n-1$ ways to choose the vertex of degree one, and the first $i+2$ vertices (including the first vertex chosen for the reverse direction) are uniquely determined once the starting vertex of degree one has been chosen. For each of the remaining $k-i-1$ vertices, there are two choices. So,

$$P^{(n)}{}_{13\cdots33} = 4\cdot 2^{n-1}\cdot \sum_{i=k/2+1}^{n-1} 2^{k-i-1} = 2^{\frac{k}{2}+n} - 2^{k+1}$$

With a similar discussion as in (VI.1) and (VI.2), respectively, we have the following two formulas when k is odd.

(VI.3) k is odd and $1 \le k \le n-1$

$$P^{(n)}{}_{13\cdots33} = 4\cdot 2^{n-1}\cdot 2^{\frac{k-1}{2}} = 2^{\frac{k+1}{2}+n}$$

(VI.4) k is odd and $n \le k \le 2n-3$

$$P^{(n)}{}_{13\cdots33} = 4\cdot 2^{n-1}\cdot \sum_{i=(k+1)/2}^{n-1} 2^{k-i-1} = 2^{\frac{k+1}{2}+n} - 2^{k+1}$$

VII. $i_1 i_2 \cdots i_{k+1} = \underbrace{3\cdots3}_{k+1}$

(VII.1) k is even. Such a path exists if and only if $k \le 2n-4$, that is $n \ge k+4$

If $k = 2n-4$, i.e. $n = (k+4)/2$, by the recursion of D[n], such a path corresponds to a path of type 13···31 of length k in D[n−1]. Hence, by **I**

$$P^{(n)}{}_{13\cdots33} = P^{(\frac{k+4}{2})}{}_{33\cdots33} = P^{(\frac{k+4}{2}-1)}{}_{13\cdots31} = 2^{\frac{k}{2}+\frac{k+4}{2}-1-1} = 2^{k}, (k = 2n-4)$$

If $k < 2n-4$, i. e. $n > (k+4)/2$, by the recursion of D[n], a 3···3 path in D[n] is either a 3···3 path in D[n−1], or a 13···31 path in D[n−1], or a 13···3 path in D[n−1]. So,

$$P^{(n+1)}{}_{33\cdots33} = P^{(n)}{}_{33\cdots33} + P^{(n)}{}_{13\cdots31} + P^{(n)}{}_{13\cdots33} \qquad (1)$$

Using (1) recursively, and by **I** and **VI**, we have

$$P^{(n)}{}_{33\cdots 33} = P^{(\frac{k+4}{2})}{}_{33\cdots 33} + \sum_{i=\frac{k+4}{2}}^{n-1} (P^{(i)}{}_{13\cdots 31} + P^{(i)}{}_{13\cdots 33})$$

$$= \begin{cases} 2^k + \sum_{i=\frac{k+4}{2}}^{n-1} (3\cdot 2^{\frac{k}{2}+i-1} - 2^{k+1}), k \ge n \\ 2^k + \sum_{i=\frac{k+4}{2}}^{k} (3\cdot 2^{\frac{k}{2}+i-1} - 2^{k+1}) + \sum_{i=k+1}^{n-1} 3\cdot 2^{\frac{k}{2}+i-1}, k \le n-1 \end{cases}$$

$$= \begin{cases} 3\cdot 2^{n+\frac{k}{2}-1} - (2n+1-k)\cdot 2^k, n \le k \le 2n-4 \\ 3\cdot 2^{n+\frac{k}{2}-1} - (3+k)\cdot 2^k, k \le n-1 \end{cases}$$

(**VII.2**) k is odd. Such paths exist if and only if $k \le 2n-5$, that is $n \ge (k+5)/2$.
If $k = 2n-5$ and $k \ge 3$, such a path can be considered as a path of type 13···3 of length k in D[n−1]. So, by **(VI.4)**,

$$P^{(n)}{}_{3\cdots 3} = P^{(\frac{k+5}{2})}{}_{3\cdots 3} = P^{(\frac{k+5}{2}-1)}{}_{13\cdots 3} = 2^{\frac{k+1}{2}+\frac{k+5}{2}-1} - 2^{k+1} = 2^{k+1}$$

If $3 \le k < 2n - 5$ i.e. $n \ge (k+5)/2 + 1 \ge 5$, such a path corresponds to either a path of type 3···3 in D[n−1] or a path of type 13···3 in D[n−1]. So

$$P^{(n)}{}_{3\cdots 3} = P^{(n-1)}{}_{3\cdots 3} + P^{(n-1)}{}_{13\cdots 3} \qquad (2)$$

Applying (2) recursively and by **VI**,

$$P^{(n)}{}_{3\cdots 3} = P^{(\frac{k+5}{2})}{}_{3\cdots 3} + \sum_{i=\frac{k+5}{2}}^{n-1} P^{(i)}{}_{13\cdots 3}$$

$$= \begin{cases} 2^{k+1} + \sum_{i=\frac{k+5}{2}}^{n-1} (2^{\frac{k+1}{2}+i} - 2^{k+1}), n \le k \le 2n-5 \\ 2^{k+1} + \sum_{i=\frac{k+5}{2}}^{k} (2^{\frac{k+1}{2}+i} - 2^{k+1}) + \sum_{i=k+1}^{n-1} 2^{\frac{k+1}{2}+i}, 3 \le k \le n-1 \end{cases}$$

$$= \begin{cases} 2^{n+\frac{k+1}{2}} - (2n+1-k)\cdot 2^k, n \le k \le 2n-5 \\ 2^{n+\frac{k+1}{2}} - (3+k)\cdot 2^k, 3 \le k \le n-1 \end{cases}$$

If k = 1 and n = 3, such a path can be considered as a path of type 13 in D[2]. By **(VI.3)**,

Applying (2) recursively and again by **VI**,

$$P^{(3)}{}_{33} = 2^3$$

$$P^{(n)}{}_{33} = P^{(3)}{}_{33} + \sum_{i=3}^{n-1} P^{(i)}{}_{13}$$

$$= \begin{cases} 2^{n+1} - 8, n \geq 3 \\ 0, n = 1, 2 \end{cases}$$

$$= 2^3 + 2^4 + \cdots + 2^n$$

Therefore, we have

$$P^{(n)}{}_{3\cdots3} = \begin{cases} 2^{n+\frac{k+1}{2}} - (2n+1-k)\cdot 2^k, n \leq k \leq 2n-5 \\ 2^{n+\frac{k+1}{2}} - (3+k)\cdot 2^k, 1 \leq k \leq n-1 \end{cases}$$

VIII. $i_1 i_2 \cdots i_{k+1} = \underbrace{3\cdots3}_{k_1} 4 \underbrace{3\cdots3}_{k_2}$, where $k_1 + k_2 = k$, $k_1 \geq 1$, $k_2 \geq 1$

By the symmetry of k_1 and k_2, we may assume $k_1 \geq k_2$.

(VIII.1) k is even

If $k_1 = k_2 = k/2$, such a path can be seen as a path of type 13···343···31 in D[k/2].

By **IV**,

$$P^{(n)}{}_{3\cdots343\cdots3} = P^{(k/2)}{}_{13\cdots343\cdots3} = 3\cdot 2^{k-1}$$

If $k_1 > k_2$, that is $k_1 \geq k/2 + 1$, such a path can be seen as a path of type 13···343···3 in D[k_1]. By **V**,

If $k \leq n$, then

$$P^{(n)}{}_{3\cdots343\cdots3} = P^{(k_1)}{}_{13\cdots343\cdots3} = 3\cdot 2^k$$

$$\begin{aligned} P^{(n)}{}_{3\cdots343\cdots3} &= P^{(k/2)}{}_{13\cdots343\cdots31} + \sum_{k_1=k/2+1}^{k-1} P^{(k_1)}{}_{13\cdots343\cdots3} \\ &= 3\cdot 2^{k-1} + \sum_{k_1=k/2+1}^{k-1} 3\cdot 2^{k_1} \\ &= 3(k-1)2^{k-1} \end{aligned}$$

If $n+1 \leq k \leq 2n-2$, then

$$\begin{aligned} P^{(n)}{}_{3\cdots343\cdots3} &= P^{(k/2)}{}_{13\cdots343\cdots31} + \sum_{k_1=k/2+1}^{n-1} P^{(k_1)}{}_{13\cdots343\cdots3} \\ &= 3\cdot 2^{k-1} + \sum_{k_1=k/2+1}^{n-1} 3\cdot 2^{k_1} \\ &= 3(2n-k-1)2^{k-1} \end{aligned}$$

(VIII.2) k is odd

Similarly as in **(VIII.1)** (the only difference is $k_1 \neq k_2$ in this case), we have

$$\begin{aligned} P^{(n)}{}_{3\cdots343\cdots3} &= \sum_{k_1=(k+1)/2}^{k-1} P^{(k_1)}{}_{13\cdots343\cdots3} = \sum_{k_1=(k+1)/2}^{k-1} 3\cdot 2^{k_1} \\ &= 3(k-1)2^{k-1} \end{aligned}$$

and

$$\begin{aligned} P^{(n)}{}_{3\cdots343\cdots3} &= \sum_{k_1=(k+1)/2}^{n-1} P^{(k_1)}{}_{13\cdots343\cdots3} = \sum_{k_1=(k+1)/2}^{n-1} 3\cdot 2^{k_1} \\ &= 3(2n-k-1)2^{k-1} \end{aligned}$$

Based on the above computations, we can get the formula of the k-connected index of D[n] for any nonnegative integer k.

Theorem 2.1 Given a positive integer n, the k-connected index of D[n] for any nonnegative integer k are listed in Table 1.

Proof: If k = 0, let ni denote the number of vertices of degree i in D[n], then $n_1 = 2^{n+1}$, $n_3 = 2^2 + \cdots + 2^n = 2^{n+1} - 4$, and $n_4 = 1$ from the definition of D[n]. Hence

$${}^0\chi(D[n]) = \frac{2^{n+1}}{\sqrt{1}} + \frac{2^{n+1}-4}{\sqrt{3}} + \frac{1}{\sqrt{4}} = 3^{-\frac{1}{2}}(2^{n+1}-4) + 2^{n+1} + 2^{-1}$$

If $1 \leq k \leq n-1$ and k is even, then the possible types of all paths of length k in D[n] are 13···31, 3···34, 13···3, 3···3 and 3···343···3. By **I, III, (VI.1), (VII.1)** and **(VIII.1)**,

$P^{(n)}{}_{13\cdots31} = 2^{\frac{k}{2}+n-1}, P^{(n)}{}_{33\cdots34} = 2^{k+1}, P^{(n)}{}_{33\cdots33} = 3\cdot 2^{\frac{k}{2}+n-1} - (3+k)\cdot 2^k, P^{(n)}{}_{13\cdots33} = 2^{\frac{k}{2}+n}$ and $P^{(n)}{}_{3\cdots4\cdots3} = 3(k-1)\cdot 2^{k-1}$, respectively. So,

$$\begin{aligned} {}^k\chi(D[n]) &= 2^{n+\frac{k}{2}-1}\frac{1}{\sqrt{3^{k-1}}} + 2^{n+\frac{k}{2}}\frac{1}{\sqrt{3^k}} + 2^{k+1}\frac{1}{\sqrt{4\cdot 3^k}} \\ &+[3\cdot 2^{n+\frac{k}{2}-1} - (3+k)\cdot 2^k]\frac{1}{\sqrt{3^{k+1}}} + 3(k-1)\cdot 2^{k-1}\frac{1}{\sqrt{4\cdot 3^k}} \\ &= \frac{\sqrt{3}+1}{\sqrt{3^k}}2^{n+\frac{k}{2}} + \frac{(3\sqrt{3}-4)k+(\sqrt{3}-12)}{\sqrt{3^{k+1}}}2^{k-2} \end{aligned}$$

If $1 \le k \le n-1$ and k is odd, then the possible types of all paths of length k in D[n] are 3···34,13···3, 3···3 and 3···343···3 By **III**, **(VI.3)**, **(VII.2)** and **(VIII.2)**, $P^{(n)}{}_{33\cdots34} = 2^{k+1}$, $P^{(n)}{}_{13\cdots33} = 2^{\frac{k+1}{2}+n}, P^{(n)}{}_{33\cdots33} = 2^{\frac{k+1}{2}+n} - (3+k)\cdot 2^k$ and $P^{(n)}{}_{3\cdots4\cdots3} = 3(k-1)\cdot 2^{k-1}$, respectively. So,

$$\begin{aligned} {}^k\chi(D[n]) &= 2^{k+1}\frac{1}{\sqrt{4\cdot 3^k}} + 2^{n+\frac{k+1}{2}}\frac{1}{\sqrt{3^k}} + \\ &+[2^{n+\frac{k+1}{2}} - (3+k)\cdot 2^k]\frac{1}{\sqrt{3^{k+1}}} + 3(k-1)\cdot 2^{k-1}\frac{1}{\sqrt{4\cdot 3^k}} \\ &= \frac{\sqrt{3}+1}{\sqrt{3^{k+1}}}2^{n+\frac{k+1}{2}} + \frac{(3\sqrt{3}-4)k+(\sqrt{3}-12)}{\sqrt{3^{k+1}}}2^{k-2} \end{aligned}$$

The other formulas can be verified similarly.

Table 1: formula of k-connected index of D[n]

k		${}^k\chi(D[n])$
k = 0		$\frac{\sqrt{3}+1}{\sqrt{3}}2^{n+1} + \frac{\sqrt{3}-8}{2\sqrt{3}}$
$1 \le k \le n-1$	odd	$\frac{\sqrt{3}+1}{\sqrt{3^{k+1}}}2^{n+\frac{k+1}{2}} + \frac{(3\sqrt{3}-4)k+(\sqrt{3}-12)}{\sqrt{3^{k+1}}}2^{k-2}$
	even	$\frac{\sqrt{3}+1}{\sqrt{3^k}}2^{n+\frac{k}{2}} + \frac{(3\sqrt{3}-4)k+(\sqrt{3}-12)}{\sqrt{3^{k+1}}}2^{k-2}$
k = n	odd	$\frac{\sqrt{3}+1}{\sqrt{3^{k+1}}}2^{\frac{3k+1}{2}} + \frac{(3\sqrt{3}-4)k+(8-11\sqrt{3})}{\sqrt{3^{k+1}}}2^{k-2}$
	even	$\frac{\sqrt{3}+1}{\sqrt{3^k}}2^{\frac{3}{2}k} + \frac{(3\sqrt{3}-4)k+(8-11\sqrt{3})}{\sqrt{3^{k+1}}}2^{k-2}$
$n+1 \le k \le 2n-4$	odd	$\frac{\sqrt{3}+1}{\sqrt{3^{k+1}}}2^{n+\frac{k+1}{2}} + \frac{(6\sqrt{3}-8)n-(4-3\sqrt{3})k+(14-11\sqrt{3})}{\sqrt{3^{k+1}}}2^{k-2}$

	even	$\frac{\sqrt{3}+1}{\sqrt{3^k}}2^{n+\frac{k}{2}}+\frac{(6\sqrt{3}-8)n-(4-3\sqrt{3})k+(14-11\sqrt{3})}{\sqrt{3^{k+1}}}2^{k-2}$
k = 2n − 3		$\frac{3\sqrt{3}+7}{\sqrt{3^k}}2^{k-1}$
k = 2n − 2		$\frac{3\sqrt{3}+10}{\sqrt{3^{k-1}}}2^{k-2}$
k = 2n − 1		$\frac{1}{\sqrt{3^{k-3}}}2^{k-1}$
k = 2n		$\frac{1}{\sqrt{3^{k-4}}}2^{k-2}$
k > 2n		0

References

[1] M. B. Ahmadi, M. Sadeghimehr, Dig. J. Nanomater. Bios., **4**(4), 639(2009).
[2] O. Araujo and J. A. de la Pe˜na, Lin. Alg. Appl., **283**(3), 171(1998).
[3] E. Buhleier, W. Wehner, F. V¨ogtle, Synthesis, 2(2), 155(1978).
[4] C. J. Hawker, J. M. J. Fr´echet, J. Am. Chem. Soc., **112**(21), 7638(1990).
[5] W. Jiang, S. Zhang, Computing the Connectivity Indices of α- and β-Cyclodextrins, presented at the IEEE International Conference on Bioinformatics and Biomedicine Workshops, December 18-th, 2010, Hong Kong, China
[6] L. B. Kier, W. J. Murray, M. Randic, L. H. Hall, J. Pharm. Sci., **65**(8), 1226(1976).
[7] Hao Li and Mei Lu, MATCH Commun. Math. Comput. Chem., **54**(2), 417(2005).
[8] D. A. Morales and O. Araujo, J. Math. Chem., **13**(1), 95(1993).
[9] Z. Mihalic and N. Trinajstic, J. Chem. Educ., **69**(9), 701(1992).
[10] M. Randic, J. Am. Chem. Soc., **97**(23), 6609(1975).
[11] J. Rada and O. Araujo, Discr. Appl. Math., **119**(3), 287(2002).
[12] M. Randic, P. J. Hansen, P. C. Jurs, J. Chem. Inf. Comput. Sci., **28**(2), 60(1988).
[13] V. K. Singh, V. P. Tewari, D. K. Gupta and A. K. Srivastava, Tetrahedron, **40**(15), 2859(1984).
[14] D. A. Tomalia, H. Baker, J. Dewald, M. Hall, G. Kallos, S. Martin, J. Roeck, J. Ryder and P. Smith, Polym. J., **17**(1), 117(1985).